Unique Paper Code   :   **32371602**

Name of the Paper   :   **Multivariate Analysis and Nonparametric Methods**

Name of the Course   :   **B.Sc. (H) Statistics Under CBCS**

Semester   :   **VI**

Duration   :   **3 hours**

Maximum Marks   :   **75 Marks**

<u>**Instructions for Candidates:**</u>

Attempt any 4 questions from the 6 questions. All questions carry equal marks. Use of simple calculators is allowed.

1. Consider the random variables X and Y such that,

$$(X, Y) \sim BVN(0, 0, 2^2, 5^2, 0.5)$$

Obtain the distribution of (U,V) where $U = 2X + 5Y$ and $V = X/2 - Y$. Obtain both the lines of regression of X on Y and Y on X. Compute $E(X^2 Y^2)$.

2. Apply SPRT procedure for testing $H_0 : \theta = 0.5$ against $H_1 : \theta = 0.6$ when the sample has been drawn from the Binomial distribution with parameters $n = 15 \ and \ \theta$. Use $\alpha = 0.2$ and $\beta = 0.4$ to determine A and B. Obtain the decision lines and discuss the procedure for obtaining them. Find the expression for its OC and ASN function.

3. A firm computes the performance score of 15 employees belonging to its 3 different departments based on a survey. The findings are tabulated below:

Manufacturing	55	60	70	44	72
Administration	45	23	40	34	30
Sales	66	55	41	23	70

Apply an appropriate non parametric test to check if there is significant difference between the average performance of workers of the three departments of the firm. State the underlying assumptions of the test used. Furthermore, state the null and alternate hypotheses and derive the inference at 5% level of significance. (Use the critical value of KW statistic: 5.78 at 5% level of significance)

4. Let $\underline{X} = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{pmatrix}$ be a random vector that follows $N_5(\underline{\mu}, \Sigma)$ where

$$\underline{\mu} = \begin{pmatrix} 3 \\ 5 \\ 8 \\ 7 \\ 6 \end{pmatrix} \quad and \quad \Sigma = \begin{bmatrix} 6 & -1 & 1 & 2 & -3 \\ -1 & 6 & 0 & 5 & 3 \\ 1 & 0 & 3 & 1 & 5 \\ 2 & 5 & 1 & 14 & 5 \\ -3 & 3 & 5 & 5 & 14 \end{bmatrix}.$$

Answer the following:

(i)   If $\underline{X}^{(1)} = \begin{pmatrix} X_1 \\ X_3 \\ X_4 \end{pmatrix}$ and $\underline{X}^{(2)} = \begin{pmatrix} X_2 \\ X_5 \end{pmatrix}$ then partition Σ into Σ_{11} , Σ_{12} , Σ_{21} and

Σ_{22} matrices. Obtain the conditional distribution of $\underline{X}^{(1)}$ given $\underline{X}^{(2)} = \begin{pmatrix} 7 \\ 9 \end{pmatrix}$.

(ii)   Define random variables Y_1, Y_2, Y_3 as
$$Y_1 = 2X_1 + 3X_2 + X_4,$$
$$Y_2 = 3X_2 + 4X_4 + X_5,$$

$$Y_3 = X_1 - X_2 - 2X_4,$$

Find the distribution of $\underline{Y} = \begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix}$. Obtain the $Var(Y_2 + 3Y_3)$.

5. In order to determine whether the median lifespan of certain species of animal is greater than 5 years, the following random sample of life span of 25 animals (in years) was obtained:

   11.3, 5.8, 3.1, 4.1, 7.3, 4.4, 1.4, 2.5, 6.6, 7.6, 24.9, 30.1, 2.9, 5.5, 7.2, 3.2, 3.9, 7.2, 20.1, 3.1, 6.1, 4.9, 19.4, 4.2, 6.3

   Describe the non-parametric test that you will use to test whether the average lifespan is greater than 5 years? State the underlying assumptions of this test. Apply the test at 5% level of significance and comment on the result.

6. Consider the random variables X_1, X_2, X_3 and X_4 which are measured from their respective means and have standard deviations as 12, 13, 8 and 14 respectively.
   Further, it is given that
   $r_{12} = 0.609$, $r_{13} = 0.693$, $r_{14} = -0.08$,
   $r_{23} = 0.946$, $r_{24} = 0.525$, $r_{34} = 0.459$
   and the value of the determinant of correlation matrix (ω) = 0.023828.
   Obtain the value of $R^2_{4.123}$ and interpret the result. Also, obtain the equation of plane of regression of X_4 on X_1, X_2 and X_3.