8 (a) Discuss Kruskal-Wallis test. How do you proceed if there are ties in the samples?

(b) The green yield (in kg) under four treatments is tabulated below:

No. of Plots	Treatments			
	1	2	3	4
1	3.16	3.43	3.15	2.48
2	3.38	2.88	2.70	2.38
3	3.49	2.97	3.10	2.57
4	2.86	3.28	2.81	2.85
5	3.87	3.95	3.45	3.00
6	4.00	3.86		2.47
7	3.61	3.26		

Test the hypothesis of equality of treatments' effect using an appropriate non parametric test at 5% level of significance. ($\chi^2_{0.05,3}$=7.815)

6,6

9 (a) Let X_1, X_2 .... be the *iid* sequence of random variables from normal distribution with mean μ and variance 144. In order to test $H_0: \mu = 48$ against $H_1: \mu = 52$, obtain the SPRT procedure of strength (α, β). Further, if $\alpha = 0.2$, $\beta = 0.4$,obtain A and B. Complete the following table related to SPRT:

m	Random observation (x_i)	a_m	$S_m = \sum_{i=1}^{m} x_i$	r_m	Decision at m[th] stage
1	50.1697				
2	48.3278				
3	54.3337				
4	22.0001				

Also represent the SPRT for the above case graphically.
(ln(A) = 1.099 and ln(B) = -.693)

(b) Twelve 6[th] grade boys, who are underweight, are put on a special diet for one month. Each boy is weighed before and after the one month dietary plan. Following table was obtained:

Before	65	63	71	60	66	72	78	74	58	59	77	65
After	70	68	75	60	69	70	81	81	66	56	79	71

Using a suitable non parametric test, can you conclude that the dietary plan has been effective at 5% level of significance? Set up the (null and alternative hypotheses.

6,6

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1363 A

Unique Paper Code : 32371602

Name of the Paper : Multivariate Analysis and Non-parametric Methods

Name of the Course : **B.Sc. (H) Statistics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question **1** is compulsory.
3. Attempt **five** more questions selecting **three** questions from **Section I** and **two** questions from **Section II**.
4. Use of simple calculators is allowed.

1. Compulsory Question

(a) Fill in the blanks

(i) The non-parametric test which is applicable for variable measured in nominal scale is ....

(ii) The Proportion of variance accounted by the first two principal components is given by ....

(iii) The limits of partial correlation coefficients are ....

(iv) In the case of SPRT, if $\frac{1-\beta}{\alpha} \le \frac{L_{1m}}{L_{om}} \le \frac{\beta}{1-\alpha}$ then we ....

(v) If $(X, Y) \sim BVN(0, 0, 4, 16, 0.8)$ then the distribution of $Z = 4X + 7Y$ is ....

1 ×5=5

(b) (i) Find the number of runs and the length of each run in the following data: ABBBBAABAAABBAAAABABB

(ii) If sample observations on vectors $\underset{\sim}{X}_1$ and $\underset{\sim}{X}_2$ is given by $\underset{\sim}{x}'_1 = (10.5, 12.3, 14.4, 16.9, 17.8)$ and $\underset{\sim}{x}'_2 = (20.6, 23.4, 21.0, 22.0, 26.5)$ respectively then obtain sample variance-covariance matrix S.

2×2=4

(c) (i) Determine the parameters for the BVN distribution with pdf $f(x,y) = k\,exp[-4x^2 - 6xy - 9y^2]$. $(x,y) \in R^2$
Hence calculate the value of k.

(ii) For a SPRT of strength (α_1, β_1) show that
$\alpha_1 \le \frac{\alpha}{1-\beta}$ and $\beta_1 \le \frac{\beta}{1-\alpha}$, given $A \approx \frac{1-\beta}{\alpha}$ and $B \approx \frac{\beta}{1-\alpha}$.
Hence show that $\alpha_1 + \beta_1 \le \alpha + \beta$.

3×2=6

Section I

2 (a) If $(X, Y) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ then prove that the marginal pdf's of X and Y are normal. However, the converse is not true.

(b) If $(X, Y) \sim BVN(0, 0, 1, 1, \rho)$ then show that

(i) $X+Y$ and $X-Y$ are independently distributed.

(ii) $Q = \frac{X^2 - 2\rho XY + Y^2}{1-\rho^2}$ is distributed as χ^2- variate.

6,6

3 (a) Let $f(x)$ and $g(y)$ be the pdf's of the random variables X and Y, respectively with corresponding cdf's $F(x)$ and $G(x)$. Also let

$$h(x,y) = f(x)g(y)[1 + \alpha\{2F(x) - 1\}\{2G(y) - 1\}]; \ |\alpha| \le 1,$$

where α is a constant and h(x,y) is a [redacted] joint pdf with marginal pdfs $f(x)$ and $g(y)$. If $f(x)$ and $g(y)$ represents standard normal distributions then prove that $Cov(X, Y) = \frac{\alpha}{\pi}$.

(b) Let $\underset{\sim}{X} \sim N_3(\underset{\sim}{0}, \Sigma)$ with $\Sigma = \begin{pmatrix} 1 & \rho & 0 \\ \rho & 1 & \rho \\ 0 & \rho & 1 \end{pmatrix}$. Show that $\rho^2 < \frac{1}{2}$. Further, obtain the value of ρ such that $(X_1 + X_2 + X_3)$ and $(X_1 - X_2 - X_3)$ are independent.

6,6

4 (a) Assuming that X_1, X_2, X_3 and X_4 are measured from their respective means, obtain the equation of plane of regression of X_3 on X_1, X_2 and X_4.

(b) What are Principal Components? Discuss the properties of Principal Components. How do you obtain the Principal Components and their variances? What are it's uses?

6,6

5 (a) If $\underset{\sim}{X} \sim N_p(\underset{\sim}{\mu}, \Sigma)$ then prove that the marginal distribution of sub-vectors $\underset{\sim}{X}^{(1)}$ and $\underset{\sim}{X}^{(2)}$ having q and $(p\text{-}q)$ different components of $\underset{\sim}{X}$ respectively, are multivariate normal.

(b) Consider the following factor model:

$$X_1 - \mu_1 = 0.5F_1 + 0.5F_2 + \epsilon_1$$
$$X_2 - \mu_2 = 0.3F_1 + 0.3F_2 + \epsilon_2$$
$$X_3 - \mu_3 = 0.5F_1 - 0.5F_2 + \epsilon_3;$$

with $\psi = Cov(\underset{\sim}{\epsilon}) = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

Compute the variance-covariance matrix Σ associated with random vector $\underset{\sim}{X}' = (X_1, X_2, X_3)$. Also find out the communalities and hence show the decomposition of variances into communalities and specific variances.

7,5

Section II

6 (a) Let $X_1, X_2, \ldots, \ldots$ be iid sequence of random variables with pdf/pmf $f(x;\ \theta)$. Describe SPRT procedure for testing $H_o: \theta = \theta_o$ against $H_1: \theta = \theta_1$. Also define OC function and ASN function for the same.

(b) Construct SPRT for testing $H_o: \sigma^2 = \sigma_o^2$ against $H_1: \sigma^2 = \sigma_1^2\ (> \sigma_o^2)$ on the basis of a random sample drawn from $N(\mu, \sigma^2)$, where μ is known. Also obtain its OC and ASN functions.

(5,7)

7 (a) Discuss Kolmogorov-Smirnov test in detail. Mention the assumptions for which this test holds.

(b) The following data represents the two independent samples:

X	47	46	32	41	40	49	50	31	52	34		
Y	45	42	59	48	56	53	48	71	43	55	33	37

Using a suitable non-parametric test, check whether the two samples are drawn from the same population at 5% level of significance? Set up the null and alternative hypotheses. (tabulated $U_{10,12}(0.05) = 29$).

6,6