Unique paper Code	:	32371303
Name of the Paper	:	Mathematical Analysis
Name of the Course	:	B.Sc. (H) Statistics Under CBCS
Semester	:	III
Duration	:	3 Hours
Maximum Marks	:	75

Instructions for Candidates

Attempt any four questions. All Questions carry equal marks .

1 (i) Find limit superior and limit inferior of $<a_n>$, where

$$a_n = \left(1 + \tfrac{1}{n}\right)^{n+1} \qquad n \in Z^+$$

(ii) Find $G = \bigcup_{n \in N} G_n, \quad where \;\; G_n = \left[-1 + \tfrac{1}{n},\ 1 - \tfrac{1}{n}\right],\ n \in N$. Is G a closed set? Justify.

(iii) Does the sequence $<a_n>$, where $a_n = 3 + (-1)^n$ converge? If yes, find its limit.

(iv) Is every convergent series absolutely convergent? Justify your answer

(v) Is every continuous function derivable? Justify your answer.

2(i) State Cauchy's second theorem on limits. Let a sequence $<a_n>$ is defined as $a_1 = 1,\ a_n = \frac{n}{n-1}$, $n \geq 2$. Show that $\lim_{n\to\infty} n^{1/n} = 1$

(ii) Show that the series $x - \frac{x^3}{3} + \frac{x^5}{5} - \ldots\ldots.$ converges iff $-1 \leq x < 1$

3 (i) Examine the convergence of the infinite series $\sum \frac{1}{n^2 + a^2}$

(ii) Examine continuity and derivability of

$$f(x) = x\,\frac{e^{1/x} - 1}{e^{1/x} + 1}\ , \quad x \neq 0$$

$$f(0) = 0$$

4 (i) Let f be a function defined and continuous on [a, b] and derivable on] a, b [. Then prove that there exist a point c ∈] a, b [such that

$$f(b) = f(a) + (b-a)\,f'(c)$$

Hence or otherwise prove that $|\sin x - \sin y| \leq |x - y| \quad \forall\, x, y \in R$

(ii) Obtain Maclaurin's series expansion of $f(x) = (1 + x)^m$ where m is any real number.

5 (i) Derive an expression for interpolating a value of f(x) near the top of the tabular values

(ii) Identify the following expression and derive the same by using Newton's advancing difference formula

$$y_u = y_0 + \binom{u}{1}\Delta y_0 + \binom{u}{2}\Delta^2 y_{-1} + \binom{u+1}{3}\Delta^3 y_{-1} + \binom{u+1}{4}\Delta^4 y_{-2} + \cdots. + \binom{u+k-1}{2k-1}\Delta^{2k-1} y_{-k+1} + \binom{u+k-1}{2k}\Delta^{2k} y_{-k} + \cdots\ldots$$

6 (i) Evaluate

$$\Delta^2 (1-x)(1-2x)(1-3x)$$

where the interval of differencing being unity and u_x, v_x are the function of x.

(ii) Show that $\Delta x^{(r)} = rhx^{(r-1)}$ where r being a positive integer and h being the interval of differencing.

(iii) Show that $\Delta^n f(x) = ah^n(n!)$, where f(x) is a rational integral function of the nth degree in x.

(iv) Calculate approximations to the value of $\int_0^6 \frac{dx}{1+x^2}$ by using simpson's one-third rule.