Unique Paper Code	:	32377911
Name of the Paper	:	Financial Statistics DSE-4
Name of the Course	:	B.Sc. (H) Statistics Under CBCS
Semester	:	VI
Duration	:	3 hours
Maximum Marks	:	75 Marks

<u>Instructions for Candidates</u>

1. Attempt any **four** questions.
2. All questions carry **equal marks**.
3. Give detailed description of the Spreadsheet function, if you are using any.

1. Mr.Vijay Gupta approached a bank for borrowing 3,000,000 to purchase a house. The bank manager told him that the borrowed amount of loan is to be repaid in monthly installments over 15 years with an interest rate of 0.8% per month. Further, the bank charges a processing fee of 6000, a house inspection fee of 4000, and 0.5% of the loan amount has to be paid to the bank after the amount is received. What is the effective annual interest rate of the loan being offered?

2. Consider the current price of a non-dividend paying stock is 265. If the exercise price is 250, the risk-free rate is 10% per annum and the price of a one-month European call option is 20.10, calculate the price of European put option on the same underlying, delivery price and maturity.
Further, discuss in detail with examples (of put price above or below), how the arbitrage opportunities might arise, if the put-call parity for European options does not hold.

3. (i) Define the following: Filtration, the Wiener process.
(ii) Consider a collection of independent and identically distributed stochastic variables $\{Y_1, Y_2, \dots\}$ with mean zero. Let $F_n = \sigma\{Y_1, Y_2, \dots Y_n\}$ denote the σ −algebra generated by $Y_1, Y_2, \dots Y_n$. Further, $\{F_1, F_2, \dots\}$ be the filtration. Then, is the sum $X_n = \sum_{i=1}^{n} Y_i$ a martingale?
(iii) Let $\{W_t; t \geq 0\}$ be a Wiener process. Are the following stochastic processes $\{X_t\}$ martingales?
   - $X_t = 2W_t + t$
   - $X_t = W_t^2$

4. Consider the geometric Brownian motion process
$$dS_t = \mu\, S_t dt + \sigma\, S_t\, dW_t, S_0 > 0.$$
(i) Determine the solution process to the above SDE. Also obtain the mean and variance functions.
(ii) Further, investigate dynamics of the transformed price process $G(S_t) = \ln(S_t)$. Identify the distribution of $G(S_t)$.

5. (i) Stating clearly the notations, write the Black–Scholes–Merton (BSM) formulas for call and put options price on a non-dividend-paying stock.
(ii) Investigate the limiting case of Black-Scholes Call option price as $S_t \to \infty$.
(iii) Obtain an expression for $\frac{\partial c}{\partial t}$ in the usual notations.

6. Consider that the stock price process follows the Itô process
$$dS_t = a(S_t, t)dt + b(S_t, t)dW_t$$
with $a(S_t, t) = \mu\, S_t$ and $b(S_t, t) = \sigma\, S_t$.
(i) Then calculate the 95% confidence limit for log-price $ln(S_t)$ in 3 months for a Pharma stock which has an initial price of 100 now, an expected return of 11% per annum, and a volatility of 30% per annum.
(ii) If for the same stock, current price is 220, an expected return of 25% per annum and a volatility of 45% per annum, what are the expected stock price $E[S_T]$ and variance of the stock price $var[S_T]$ in 3 months?