4. (a) Consider a strategy created by buying a call option with strike price K=$210, buying a second call option with a strike price $230 and selling two call options with a strike price $220. Construct the corresponding payoff table.

(b) A company's current stock price is $110. It is expected to increase or decrease by 20% after one period. The risk- free rate of interest for one period is 10%. Find the value of the call option at t = 0, given that the European call on the stock has a strike price of $100. $(6,6\frac{1}{2})$

Section -C

5. (a) Let W_t be a standard Wiener process. Show that the process defined as $X_t = tW_{1/t}$, for $t > 0$ is also standard Wiener process.

(b) Let W_t be a standard Wiener process. Compute $E(W_t^4)$.

$(6,6\frac{1}{2})$

6. (a) Obtain the Greek 'theta' of a European put option.

(b) State and prove Ito's lemma in general form.

$(6,6\frac{1}{2})$

7. (a) Let us consider a European put option with exercise price $K = 102$ EURO at $T = 2$. The current price of a certain stock is 100 EURO at $t = 0$. During each of the following two periods, price will either rise by 1/10 or fall by1/11. For the first period the stock price is expected to increase or decrease by 10%. For the second period, the stock price is expected to increase or decrease by 5%. Assuming the risk - free rate of interest to be 5% in each period, obtain the price of the put option at $t = 0$.

(b) Let the current price of an oil stock be $50. The risk- free rate is 3% per month and volatility associated with the stock price movement is 10% per month, find the price of a 4- month put option if the exercise price is $100. Use put call parity to determine the price of a call on this stock.

or

For a two-period binomial model, you are given:

(i) Each period is one year.

(ii) The current price for a non-dividend-paying stock is 20.

(iii) $u = 1.2840$, where u is one plus the rate of capital gain on the stock per period if the stock price goes up.

(iv) $d = 0.8607$, where d is one plus the rate of capital loss on the stock per period if the stock price goes down.

(v) The continuously compounded risk-free interest rate is 5%.

Calculate the price of an American call option on the stock with a strike price of 22. $(6,6\frac{1}{2})$

(600)

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1318 **A**

Unique Paper Code : 32377911

Name of the Paper : Financial Statistics (DSE)

Name of the Course : **B.Sc. (H) Statistics under**

Semester : VI

Duration : 3 + (additional ½ hour) Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.
3. **Section A** is compulsory.
4. Choose **two** questions from each of the **sections B** and **C**.
5. Use of non-programmable calculator is permitted.

Section A

Attempt any five parts. All parts carry equal marks. (5X5=25 marks)

1. (a) A project requires an initial investment of ₹750,000 and is expected to generate the following net cash inflows:

Year1: ₹100,000

Year2: ₹150,000

Year3: ₹200,000

Year4: ₹250,000

Year 5: ₹250,000

Calculate the net present value of the project if minimum desired rate of return is 10%.

(b) Calculate the internal rate of return for the given cash flow streams of the two investments: (-2,0,5) and (-1,2). Also discuss which is a more desirable investment.

(c) Consider a strategy where an investor buys one stock, one European put with an exercise price K and sells one European call with an exercise price K. Calculate the corresponding payoff and explain the risk of this strategy.

(d) Let W_t be a standard Wiener process. Define $X_t = W_t^3 - 3tW_t$. Show that X_t is a martingale with respect to F_t, the filtration associated with W_t

(e) Suppose that two assets in the portfolio have expected rate of return are $\bar{r}_1$ and $\bar{r}_2$. The respective variances and covariance are σ_1, σ_2 and σ_{12}. Construct a portfolio consisting of the two stocks so that it has the minimum variance in the class of all portfolios consisting of these two assets. What is the mean rate of return of this portfolio?

(f) Consider a forward contract on a non-dividend paying share for which the stock price process is an Ito-process. If 'r' is the risk-free rate of interest, then find the SDE for the contract F maturing at time T.

(g) Consider a strategy involving a European long call option with a certain strike price K_1 and a European short call option on the same stock with a relatively higher strike price K_2. Both the call options have the same maturity date. Draw the corresponding payoff table.

(h) An investor sold 2000 calls and 1500 puts with the same exercise price and same maturity on the same underlying stock. Delta for the call is 0.42 and delta for put is -0.38. How many stocks should the investor sell or buy in order to have a delta neutral position.

(i) $1000 is kept in a savings account at 3% compounded continuously for 3 years. It is then withdrawn and placed in another bank account at a rate of 4% compounded continuously for 5 years. What will be the balance in the second account after 5 years?

(j) Consider two 3-year bonds, Bond A has a 10% coupon and sells for ₹98 and Bond B has an 8% coupon and sells for ₹93.20. Both the bonds have same face value, normalized to 100. Obtain the 3-year spot rate of a constructed zero-coupon portfolio.

Section-B

2. (a) Consider a long forward contract to buy a stock which has a price of S_t at time t. Let K be the delivery price and T be the maturity date. Further let $V(S_t, \tau)$ denote the value of the long forward contract at time t. The time to maturity is $\tau = T - t$. Assume that the interest rate is constant during the time to maturity. The stock does not pay any dividend and does not involve any costs during the time to maturity. Obtain the value of the forward contract and the forward price for the contract.

(b) The cash flows of a 3-year project are:

- an initial outlay of £35,000
- an income of £12,000 *p.a.* during the first year (assumed to be payable continuously)
- an income of £15,000 *p.a.* each during the next two year (assumed to be payable continuously)
- regular expenditure of £2,500 *p.a.* during the first 2 years (assumed to be payable continuously)
- a decommissioning expense of £30,000 at the end of the 3th year.

Calculate the IRR of this project. $(6,6\frac{1}{2})$

3. (a) For two European puts with the same maturity date T and delivery prices $K_1 \leq K_2$ it holds at time $t \leq T$:

$$0 \leq P_{K_2,T}(S_t, \tau) - P_{K_1,T}(S_t, \tau) \leq (K_2 - K_1)e^{-r\tau}$$

with $\tau = T - t$, denoting time to maturity and r the inherent rate. Further if option prices are differentiable as a function of delivery price, then in the limiting case $K_2 \to K_1$ it follows

$$0 \leq \frac{\partial P}{\partial K} \leq e^{-r\tau} \leq 1.$$

or

The correlation (ρ) between two assets 'A' and 'B' is 0.15. The expected returns and standard deviations of returns are given in the following table:

Asset	Return (%)	Standard deviation (%)
A	12	12
B	20	25

Find the proportions a and $(1-a)$ of 'A' and 'B' respectively that defines a portfolio consisting of them such that it has minimum variance.

Find the expected return and the standard deviation of this portfolio.

(b) Let $\{X_t : t > 0\}$ denote a geometric random walk. Prove that the process $\{Y_t = \log(X_t) ; t > 0\}$ is a binomial process.

$(6,6\frac{1}{2})$