Unique paper Code	:	32371601
Name of the Paper	:	Design of Experiments
Name of the Course	:	B.Sc. (H) Statistics under CBCS
Semester	:	VI
Duration	:	3 hours
Maximum Marks	:	75

Instructions for Candidates

Attempt *four* questions in all. All questions carry equal marks.

Use of simple calculators is allowed.

1. An engineer wants to study the effect of three different lubricating oils (A, B, C) on fuel economy in diesel truck engines (1, 2, 3, 4, 5). He conducts an experiment by using five samples of each kind of lubricating oil. The samples are tested in a completely random order.

TRUCK				
1	2	3	4	5
B	C	A	C	A
C	B	B	A	B
C	A	B	A	C

Write the appropriate hypothesis along with the alternative hypothesis, the underlying model, assumptions for errors and additional constraints for the given design, say D1. To test the effect of lubricating oils, he further decides to conduct the experiment using five different types of truck engines, subject to the condition that each oil is allocated randomly to each truck engine.

TRUCK				
1	2	3	4	5
C = 0.50	B = 0.63	A = 0.48	B = 0.33	A = 0.51
A = 0.53	A = 0.66	C = 0.52	C = 0.43	B = 0.54
B = 0.51	C = 0.59	B = 0.49	A = 0.4	C = 0.51

Do you think there will be a change in the design to be used now? If yes, name the new design. Write the underlying model, any additional hypothesis that you can test now along with the additional constraints for the newly identified design D2. Arrange the data according to the layout of the new design. Discuss the advantages and draw-backs of using design D2 instead of design D1.

2. Show that the matrix N given below is the incidence matrix of a BIBD. Obtain the parameters of this design and generate it.

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

Define complementary, residual and derived designs and obtain them for the above design along with their parameters. Also verify if the resulting designs are BIBDs.

3. (a) Complete the following table for the analysis of variance of a fixed effects Latin Square design:

Sources of Variation	d.f.	Sum of Squares	Mean Squares	F
Columns	5	-	-	
Rows	-	4.2	-	
Treatments	-	-	2.43	-
Errors	-	-	0.65	
Total	-	39.65		

(b) Suppose we have v treatments to be compared in v^2 plots. Name the design that you will carry out under each of the following situations:

(i) There is no fertility difference among the v plots,

(ii) The fertility changes along a particular direction, and

(iii) The fertility changes along two perpendicular directions.

Give appropriate analysis of variance table for each case.

(c) Define a symmetric BIBD. Is it possible to have a symmetric BIBD with the following parameters? Give reasons.

(i) $v = b = 88,\quad r = k = 30,\quad \lambda = 10$

(ii) $v = b = 46,\quad r = k = 10,\quad \lambda = 2$

(d) The following are two key blocks of a layout plan before randomization for a 2^4 experiment with factors A, B, C and D:

Replication I: $(1), abc, abd, cd$

Replication II: $(1), abc, acd, bd$

Find the effect or effects confounded.

4. Construct a 2^5 factorial experiment with factors A, B, C, D and E in 2^2 blocks of size 2^3 each, in two replicates, confounding ABC and ADE in one replicate, and BD and ABC in the other replicate. Also state the interactions that are automatically confounded in each replicate. Construct the ANOVA table for the design obtained above.

5. The following are two blocks of a layout plan before randomization for a 3^3 factorial experiment with factors A, B and C:

Replication I: $001, 102, 010$

Replication II: $001, 100, 202$

Identify all the confounded effects. Write down the contents of the remaining blocks. Also, construct a system of partial confounding for a 3^2 factorial experiment in blocks of size 3 with 6 blocks, so that at least partial information can be obtained about two factor interaction components and full information about the main effects. Write the ANOVA table for the constructed design.

6. What do you mean by fractional factorial designs? Discuss one half fraction of the 2^4 design explaining clearly the terms generator, defining relation, aliases, alias structure, principal fraction and word. Construct a 2^{5-2} design with defining relations I = ABD and I = -BCDE. Also write the alias structure of this design and identify its resolution.