

(c) Apply modified Heun's method to calculate $y(1)$, given that

$$\frac{dy}{dx} = x + 2y; y(0) = 0; h = 0.5. \quad [6]$$

(d) Evaluate $I = \int_0^1 x\sqrt{1+x} dx$ using Trapezoidal rule with 4 subintervals. [6]

Q-6. (a) Evaluate $\int_0^\pi \sin x dx$ using Simpson rule by dividing interval into four equal parts. [6.5]

(b) Calculate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for $x = 5$, given the following table: [6.5]

x	2	4	9	10
$f(x)$	4	56	71	90

(c) Apply modified Euler's method to approximate the solution of the initial value problem and calculate $y(1.3)$ by using $h = 0.1$:

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}, y(1) = 1. \quad [6.5]$$

(d) The following table of values is given:

x	0.6	0.8	0.9	1.0	1.1	1.2	1.4
$f(x)$	0.7072	0.8599	0.9259	0.9840	1.0337	1.0746	1.1280

using the formula $f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}$, with $h = 0.4, 0.2, 0.1$ and the Richardson extrapolation, find $f'(1)$. [6.5]

[This question paper contains 4 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2779

A

Unique Paper Code : 62357603

Name of the Paper : Numerical Methods

Name of the Course : B.A. (Prog.)

Semester : VI

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All six questions are compulsory.
3. Attempt any two parts from each question.
4. Use of Non - Programmable Scientific Calculator is allowed.

Q-1. (a) Find the real root of the equation $2x^3 - 3x + 1 = 0$ by Regula Falsi method. Perform two iterations. [6]

(b) Use secant method to find root of $3x + \sin(x) - e^x = 0$ in $]0,1[$. Perform two iterations. [6]

(c) Define the floating point representation, Global error and Truncation error with examples. [6]

(d) Obtain the Rate of Convergence of Bisection method. [6]

Q-2. (a) Find the value of :-

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

with an absolute error smaller than 0.005 for $x = 0.2145E0$ using Normalized

floating point arithmetic with 4 digit mantissa.

[6.5]

(b) Evaluate the sum $S = \sqrt{3} + \sqrt{8} + \sqrt{10}$ to four significant digits and find its absolute and relative error.

[6.5]

(c) Use Bisection method to find a real root of the equation

$$f(x) = 2x - \sqrt{1 + \sin x} = 0.$$

[6.5]

(d) Find a real root of the equation $3x = \cos x + 2$ by Newton-Raphson method.

[6.5]

Q-3. (a) The function $y = f(x)$ is given at the point (7, 3), (8, 1), (9, 1) and (10, 9).

Find $f(8.5)$ using Lagrange's interpolation technique.

[6]

(b) Find the inverse of the following matrix using the Gauss-Jordan method:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

[6]

(c) For the following system of equations:

$$4x + y + 2z = 4$$

$$3x + 5y + z = 7$$

$$x + y + 3z = 3.$$

Use Gauss-Jacobi iteration method by performing three iterations. Take the initial

approximation as $(x, y, z) = (0, 0, 0)$.

[6]

(d) If $f(x) = \frac{1}{x}$ then evaluate Newton Dividend difference $f[a, b, c, d]$. Also prove

the following relation:

$$(1 - \nabla)^{-1} = 1 + \frac{\delta^2}{2} + \delta \sqrt{1 + \frac{\delta^2}{4}}$$

[6]

Q-4. (a) Solve the linear system $Ax = b$ using Gauss-Elimination method with row pivoting:

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 4 & -2 & 1 \\ 3 & -1 & 3 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix} \quad [6.5]$$

(b) Starting with initial vector $(x, y, z) = (1, 1, 1)$, perform three iterations of Gauss-Seidel method to solve the following system of equation:

$$10x + 2y + z = 9$$

$$2x + 20y - 2z = -44$$

$$-2x + 3y + 10z = 22.$$

[6.5]

(c) Find the cubic polynomial which takes the following values:

[6.5]

x	0	1	2	3
$f(x)$	1	2	1	0

(d) Apply Gauss-Jordan method to solve:

$$x + 2y + z = 8$$

$$2x + 3y + 4z = 20$$

$$4x + 3y + 2z = 16.$$

[6.5]

Q-5. (a) The velocity v (km/min) of a moped which starts from rest, is given at fixed intervals of time t (min) as follows:

[6]

t	0	2	4	6	8	10	12
v	0	10	18	25	29	32	20

Estimate approximately the distance covered in 12 minutes by Simpson's $\frac{1}{3}$ rd rule.

(b) Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$, at $y = 0.1$ by taking $h = 0.02$ by using

Euler's method.

[6]