(c) Define a conditionally convergent series. Test for the convergence and absolute convergence of the series.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n+2}{2^n+5}.$$

(d) Test for convergence the series

$$\sum_{n=1}^{\infty} \frac{1.3.5.....(2n-1)}{2.4.6.....2n} \cdot \frac{1}{n}$$

6 (a) Define Riemann integrability of a bounded function f on a bounded closed interval $[a, b]$. Show that the function f defined on $[a, b]$ as

$$f(x) = \begin{cases} 0, & \text{when } x \text{ is rational} \\ 1, & \text{when } x \text{ is irrational} \end{cases}$$

is not Riemann integrable.

(b) Test for convergence the series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cos n\alpha}{\sqrt{n^3}}, \ \alpha \text{ being real.}$$

(c) Integrate the function $f(x) = x[x]$ on $[0, 4]$, where $[x]$ denotes the greatest integer not greater than x.

(d) Show that the sequence $\langle a_n \rangle$ defined as $a_n = 1 + \frac{1}{6} + \frac{1}{11} + ... + \frac{1}{5n-4}$ is not a Cauchy sequence.

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 2792 **A**

Unique Paper Code : 62354443

Name of the Paper : Analysis (LOCF)

Name of the Course : **B.A. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.

1. (a) Find the supremum and infimum of the following sets, if they exist

(i) $E = \left\{1 + \frac{(-1)^n}{n} : n \in N\right\}$

(ii) $F = \left\{2, \frac{3}{2}, \frac{4}{3}, ..., \frac{n+1}{n}, ...\right\}$

(b) State Sequential criterion of continuity.

Define $f:\mathbb{R}\to\mathbb{R}$ by $f(x)=\begin{cases}1, & \text{when } x \text{ is rational}\\ 0, & \text{when } x \text{ is irrational}\end{cases}$.

Show that f is discontinuous on $\mathbb{R}$.

(c) Give an example of a non-empty bounded subset S of R whose supremum and infimum both belong to R ~ S.

(d) Test for convergence the series

$$\sum_{n=1}^{\infty}\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$$

2. (a) Show that $f(x)=\begin{cases}\dfrac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}, & x\neq 0\\ 1, & x=0\end{cases}$

is discontinuous at $x=0$

(b) State Archimedean Property of real numbers. Use it to prove that if $t>0$, there exists $n_t\in N$ such that $0<1/n_t<t$.

(c) Show that the function $f(x)=x^2$ is uniformly continuous on $]-2,2[$.

(d) Prove that if

$$a_n=\frac{1}{n}\{(n+1)(n+2).....(n+n)\}^{1/n}$$

then $\langle a_n\rangle$ converges to $\frac{4}{e}$.

3. (a) Prove that every Cauchy sequence is bounded but converse need not be true.

(b) Prove that the series

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+..... \text{ converges.}$$

(c) Show that the sequence $\langle s_n\rangle$ where $S_n=\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+....+\frac{1}{n!}$ is convergent.

(d) Show that $\lim_{n\to\infty}\frac{1+3+5+.....+(2n-1)}{n^2}=1$.

4 (a) Show that the series $1+r+r^2+r^3+....(r>0)$ converges if $r<1$ and divergence if $r\geq 1$.

(b) Show that the sequence $\langle a_n\rangle$ where $a_n=1+\frac{1}{3}+\frac{1}{3^2}+....+\frac{1}{3^{n-1}}$ converges. Find $\lim_{n\to\infty}a_n$?

(c) Test for convergence the series

$$\frac{x}{2\sqrt{3}}+\frac{x^2}{3\sqrt{4}}+\frac{x^3}{4\sqrt{5}}+......(x>0)$$

(d) Show that the series $\sum_{n=1}^{\infty}\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ is convergent.

5 (a) Define Alternating series of real numbers. Test for the convergence and absolute convergence of the series.

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+....$$

(b) Prove that every continuous function is integrable.