Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351201_OC**
Name of Paper	: **C 3-Real Analysis**
Semester	: **II**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

Q1. Find the infimum and supremum, if they exist, of the following subsets S_i $(i = 1,2,3)$. Justify your answer in each case:

$$S_1 = \left\{1 - \frac{(-1)^n}{n} : n \in \mathbb{N}\right\}$$
$$S_2 = \left\{\frac{\sqrt{2}}{n} : n \in \mathbb{N}\right\}$$
$$S_3 = \left\{\frac{1}{n} + 1 : n \in \mathbb{N}\right\}$$

Let S be a non-empty bounded subset of $\mathbb{R}$. Let $a \in \mathbb{R}$ and define a set $a - S = \{a - s : s \in S\}$. Prove that $a - S$ is a bounded set and $\sup(a - S) = a - \inf S$.

Q2. Let $S = \{s \in \mathbb{R} : 0 \leq s \ and \ s^2 < 3\}$. Show that the set S has a supremum in $\mathbb{R}$. If $x = \sup S$, prove that $x > 0$ and $x^2 = 3$. What is $\inf S$?

Let u and v be real numbers with $u < v$. Show that there exists a rational number r such that
$$u < \sqrt{3}\, r < v.$$

Q3. Discuss convergence or divergence of the following sequences. If convergent, find the limit of the sequence (x_n) using ϵ- definition and if divergent, give reason for the same:

(i) $x_n = \frac{2n+3}{3n+7}$ (ii) $x_n = \frac{n}{(1-n)(1+n)}$ (iii) $x_n = \frac{2^n+4^n}{3^n}$

Are these sequences bounded? Justify your answer in each case.

If (x_n) is a convergent sequence with $x_n \geq 2$ for all $n \in \mathbb{N}$, prove that $\lim(x_n) \geq 2$.

Q4. Using the definition of Cauchy sequence, establish the convergence or divergence of following sequences

(i) $\left(1 + \frac{1}{2!} + \frac{1}{3!} + \cdots - \cdots + \frac{1}{n!}\right)$ (ii) $(\ln n^2)$ (iii) $((-2)^n)$

Show that a sequence (x_n) defined as
$$x_1 = 1$$
$$x_{n+1} = \frac{x_n + 3}{5}, \qquad n \geq 1$$
is convergent and find its limit.

Q5. Check the convergence or divergence of the following series. Clearly specify the result being used:

(i) $\sum_{n=1}^{\infty} e^{-n^2}$

(ii) $\sum_{n=1}^{\infty} \frac{1}{\log n}$

(iii) $\sum_{n=1}^{\infty} \frac{n+1}{2^n}$

(iv) $\sum_{n=1}^{\infty} \left(\frac{n}{n-1}\right)^{n^2}$

Q6. For each of the following, determine whether the series converges absolutely, converges conditionally, or diverges.

(i) $\sum_{n=1}^{\infty} \frac{2^n + n}{2^n - n}$

(ii) $\sum_{n=1}^{\infty} \frac{(\sin n\alpha + cos^2 n\alpha)}{n^2}$

(iii) $\sum_{n=1}^{\infty} \frac{(-1)^n \, 100^n}{n\,!}$

(iv) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n - \log n}$