(b) Find the temperature distribution in a rod of length l. The faces are insulated, and the initial temperature distribution is given by x (l - x). (6)

(c) Establish the validity of the formal solution of the initial boundary – value problem (6)

$$
\begin{aligned}
&u_t = k u_{xx}, \quad 0 \le x \le l, t > 0, \\
&u(x,0) = f(x), \qquad 0 \le x \le l, \\
&u(0,t) = 0, \qquad t > 0, \\
&u_x(1,t) = 0, \qquad t > 0.
\end{aligned}
$$

(d) Prove the uniqueness of the solution of the problem : (6)

$$
\begin{aligned}
&u_{tt} = c^2 u_{xx}, \quad 0 < x < l, t > 0, \\
&u(x,0) = f(x), 0 \le x \le l, \\
&u_t(x,0) = g(x), 0 \le x \le l, \\
&u(0,t) = u(0,t) = 0, \quad t > 0.
\end{aligned}
$$

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1150 **A**

Unique Paper Code : 32351401_LOCF

Name of the Paper : BMATH-408 Partial Differential Equation

Name of the Course : **CBCS B.Sc. (H) Mathematics**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **four** questions.
3. **All** questions carry equal marks.

SECTION – I

Attempt any **two** parts out of the following.

Marks of each part are indicated.

1. (a) Define the following with one example each : (6)

(i) Quasi-linear first order partial differential equation (PDE).

(ii) Semi-linear first order PDE.

(iii) Linear first order PDE.

State whether the following first order PDE is quasi-linear, semi-linear, linear or non-linear :

$$(xy^2)u_x - (yx^2)u_y = u^2(x^2 - y^2)$$

Justify.

(b) Solve the Cauchy problem (6)

$$uu_x + u_y = 1$$

such that $u(s,0) = 0, x(s,0) = 2s^2$,

$$y(s,0) = 2s, s > 0.$$

(c) Obtain the solution of the pde (6)

$$x(y^2 + u)u_x - y(x^2 + u)u_y = (x^2 - y^2)u,$$

with the data u(x,y) = 1 on x + y = 0.

(c) Determine the solution of the initial-value problem (6.5)

$$\begin{aligned} &u_{tt} = c^2 u_{xx} + x^2, \\ &u(x,0) = x, && 0 \le x \le 1, \\ &u_t(x,0) = 0, && 0 \le x \le 1, \\ &u(0,t) = 0, u(1,t) = 0, && t > 0. \end{aligned}$$

(d) Determine the solution of the initial-value problem (6.5)

$$\begin{aligned} &u_t = ku_{xx}, && 0 < x < 1, t > 0, \\ &u(x,0) = x(1-x), && 0 \le x \le 1 \\ &u(0,t) = t, \quad u(1,t) = \sin t, && t > 0. \end{aligned}$$

6. Attempt any **two** parts out of the following :

(a) Determine the solution of the initial boundary-value problem by method of separation of variables (6)

$$u_{tt} = c^2 u_{xx}, 0 < x < a, t > 0$$

$$u(x,0) = 0, \ 0 \le x \le a,$$

$$u_t(x,0) = \begin{cases} \dfrac{v_0}{a} x, & 0 \le x \le a \\ v_0(l-x)/(l-a), & a \le x \le l \end{cases}$$

$$u(0,t) = 0 = u(a,t) = 0, \qquad t \ge 0.$$

$$u_{tt} = 9u_{xx}, \quad 0 < x, \infty, t > 0,$$
$$u(x,0) = 0, \quad 0 \le x < \infty,$$
$$u_t(x,0) = x^3, \quad 0 \le x < \infty$$
$$u_x(0,t) = 0, \quad t \ge 0.$$

SECTION – III

5. Attempt any **two** parts out of the following :

(a) Determine the solution of the initial boundary-value problem by method of separation of variables

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < l, \ t > 0$$
$$u(x,0) = \begin{cases} hx/a, & 0 \le x \le a \\ h(l-x)/(l-a), & a \le x \le l \end{cases},$$
$$u_t(x,0) = 0, \qquad 0 \le x \le l,$$
$$u(0,t) = 0 = u(l,t) = 0 \qquad t \ge 0$$

(6.5)

(b) Obtain the solution of IBVP (6.5)

$$u_t = u_{xx}, \qquad 0<x<2, \ t>0,$$
$$u(x,0)=x, \ 0 \le x \le 2,$$
$$u(0,t)=0, \quad u_x(2,t)=1, \qquad t \ge 0,$$

(d) Apply $\sqrt{u} = v$ and $v(x,y) = f(x) + g(y)$ to solve

$$x^4 u_x^2 + y^2 u_y^2 = 4u. \qquad (6)$$

2. Attempt any **two** parts out of the following :

(a) Apply the method of separation of variables u(x,y) = f(x)g(y) to solve

$$y^2 u_x^2 + x^2 u_y^2 = (xyu)^2$$

such that $u(x,0) = 3e^{\frac{x^2}{4}}$. (6.5)

(b) Find the solution of the equation (6.5)

$$yu_x - 2xyu_y = 2xu$$

with the condition $u(0,y) = y^3$.

(c) Reduce into canonical form and solve for the general solution (6.5)

$$u_x - yu_y - u = 1.$$

(d) Derive the one-dimensional heat equation :

$$u_t = \kappa u_{xx},$$

where κ is a constant. (6.5)

SECTION – II

3. Attempt any **two** parts out of the following :

(a) Find the characteristics and reduce the equation

$u_{xx} - (\sec h^4 x)u_{yy} = 0$ into canonical form. (6)

(b) Find the characteristics and reduce the equation

$x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} + xyu_x + y^2 u_y = 0$

into canonical form. (6)

(c) Transform the equation $u_{xx} - u_{yy} + 3u_x - 2u_y + u = 0$ to the form $v_{\xi\eta} = cv$, c=constant, by introducing the new variable $v = ue^{-(a\zeta + b\eta)}$, where a and b are undetermined coefficients. (6)

(d) Use the polar co-ordinates r and θ (x=r cosθ, y=r sinθ) to transform the Laplace equation $u_{xx} + u_{yy} = 0$ into polar form. (6)

4. Attempt any **two** parts out of the following :

(a) Find the D'Alembert solution of the Cauchy problem for one dimensional wave equation given by

$$u_{tt} - c^2 u_{xx} = 0, x \in R, t > 0$$
$$u(x,0) = f(x), x \in R,$$
$$u_t(x,0) = g(x), x \in R.$$ (6.5)

(b) Solve (6.5)

$$y^3 u_{xx} - yu_{yy} + u_y = 0,$$
$$u(x,y) = f(x) \text{ on } x + \frac{y^2}{2} = 4 \text{ for } 2 \le x \le 4,$$
$$u(x,y) = g(x) \text{ on } x - \frac{y^2}{2} = 0 \text{ for } 0 \le x \le 2,$$
$$\text{with } f(2) = g(2).$$

(c) Determine the solution of initial boundary value problem

$$u_{tt} = 16u_{xx}, \quad 0 < x < \infty, t > 0$$
$$u(x,0) = \sin x, \quad 0 \le x < \infty,$$
$$u_t(x,0) = x^2, \quad 0 \le x < \infty,$$
$$u(0,t) = 0, \quad t \ge 0.$$ (6.5)

(d) Determine the solution of initial boundary value problem (6.5)