(c) Define Maxmin and Minmax value for a Fair Game. Using Maxmin and Minmax Principle, find the saddle point, if exists, for the following pay – off matrix :

$$\text{Player 2}\overset{\text{Player 1}}{\begin{bmatrix} 1 & 3 & 6 \\ 5 & 4 & 2 \\ 2 & 1 & 3 \end{bmatrix}}.$$

(d) Convert the following Game Problem into a Linear Programming Problem for player A and player B and solve it by Simplex Method :

$$\text{Player A}\overset{\text{Player B}}{\begin{bmatrix} 2 & 3 & 1 \\ 4 & 2 & 6 \end{bmatrix}}.$$

[This question paper contains 8 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1299 **A**

Unique Paper Code : 32357616

Name of the Paper : DSE-4 Linear Programming and Applications

Name of the Course : **CBCS (LOCF) – B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Solve the following Linear Programming Problem by Graphical Method :

Minimize $3x + 2y$
subject to $5x + y \geq 10$
$x + y \geq 6$
$x + 4y \geq 12$
$x \geq 0, y \geq 0$.

(b) Define a Convex Set. Show that the set S defined as :

$$S = \{(x,y) \mid x^2 + y^2 \leq 4\} \text{ is a Convex Set.}$$

(c) Find all basic feasible solutions of the equations:

$$x_1 + x_2 + 2x_3 + 3x_4 = 12$$

$$x_2 + 2x_3 + x_4 = 8$$

(d) Prove that to every basic feasible solution of the Linear Programming Problem:

Minimize $z = cx$

subject to $Ax = b,\ x \geq 0$

there corresponds an extreme point of the feasible region.

5. (a) Solve the following cost minimization Transportation Problem :

Origin \ Destinations	I	II	III	IV	Availability
A	10	11	10	13	30
B	12	12	11	10	50
C	13	11	14	18	20
Requirements	20	40	30	10	

(b) Four new machines are to be installed in a machine shop and there are five vacant places available. Each machine can be installed at to one and only one place. The cost of installation of each job on each place is given in table below. Find the Optimal Assignment. Also find which place remains vacant.

Machine \ Place	A	B	C	D	E
I	13	15	19	14	15
II	16	13	13	14	13
III	14	15	18	15	11
IV	18	12	16	12	10

(b) State and prove the Weak Duality Theorem. Also show that if the objective function values corresponding to feasible solutions of the Primal and Dual Problem are equal then the respective solutions are optimal for the respective Problems.

(c) Using Complementary Slackness Theorem, find optimal solutions of the following Linear Programming Problem and its Dual:

Maximize $4x_1 + 3x_2$
subject to

$$x_1 + 2x_2 \leq 2$$
$$x_1 - 2x_2 \leq 3$$
$$2x_1 + 3x_2 \leq 5$$
$$x_1 + x_2 \leq 2$$
$$3x_1 + x_2 \leq 3$$
$$x_1, x_2 \geq 0.$$

(d) For the following cost minimization Transportation Problem find initial basic feasible solutions by using North West Comer rule, Least Cost Method and Vogel's Approximation Method. Compare the three solutions (in terms of the cost):

Source \ Destination	A	B	C	D	E	Supply
I	15	15	16	17	15	24
II	18	19	16	20	15	38
III	16	15	22	17	20	43
Demand	27	12	32	17	17	

2. (a) Let us consider the following Linear Programming Problem:

$$\text{Minimize } z = cx$$

$$\text{subject to } Ax = b, \ x \geq 0$$

Let $(x_B, 0)$ be a basic feasible solution corresponding to a basis B having an a_j with $z_j - c_j > 0$ and all corresponding enteries $y_{ij} \leq 0$, then show that Linear Programming Problem has an unbounded solution.

(b) Let $x_1 = 2$, $x_2 = 1$, $x_3 = 1$ be a feasible solution to the system of equations:

$$x_1 + 4x_2 - x_3 = 5$$

$$2x_1 + 3x_2 + x_3 = 18$$

Is this a basic feasible solution? If not, reduce it to two different basic feasible solutions.

(c) Using Simplex method, find the solution of the following Linear Programming Problem:

Minimize $x_1 - 3x_2 + 2x_3$
subject to $3x_1 - x_2 + 2x_3 \leq 7$

$$2x_1 - 4x_2 \geq -12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0.$$

(d) Solve the following Linear Programming Problem by Big-M method:

Maximize $x_1 - 4x_2 + 3x_3$
subject to
$$2x_1 - x_2 + 5x_3 = 40$$
$$x_1 + 2x_2 - 3x_3 \geq 22$$
$$3x_1 + x_2 + 2x_3 = 30$$
$$x_1, x_2, x_3 \geq 0.$$

3. (a) Solve the following Linear Programming Problem by Two Phase Method :

Maximize $x_1 + 4x_2 + 3x_3$
subject to
$$x_1 + x_2 + x_3 \geq 4$$
$$--2x_1 + 3x_2 - x_3 \leq 2$$
$$x_2 - 2x_3 \leq 1$$
$$x_1, x_2, x_3 \geq 0.$$

(b) Find the solution of given system of equations using Simplex Method:

$$3x_1 - 2x_2 = 8$$
$$x_1 + 2x_2 = 4$$

Also find the inverse of A where $A=\begin{bmatrix} 3 & -2 \\ 1 & 2 \end{bmatrix}$.

(c) Using Simplex method, find the solution of the following Linear Programming Problem :

Maximize $2x_1 + x_2$
subject to
$$x_1 - x_2 \leq 10$$
$$2x_1 - x_2 \leq 40$$
$$x_1 \geq 0,\ x_2 \geq 0.$$

(d) Find the optimal solution of the Assignment Problem with the following cost matrix :

Job \ Machines	I	II	III	IV	V	VI
A	4	8	5	4	6	9
B	8	3	8	4	11	7
C	9	5	7	9	8	7
D	10	9	5	6	9	9
E	5	11	9	10	10	9
F	9	5	7	10	8	7

4. (a) Find the Dual of following Linear Programming Problem :

Minimize $x_1 + x_2 + 3x_3$
subject to
$$4x_1 + 8x_2 \geq 3$$
$$7x_2 + 4x_3 \leq 6$$
$$3x_1 - 2x_2 + 5x_3 = 7$$
$x_1 \leq 0,\ x_2 \geq 0,\ x_3$ is unrestricted.