Name of Course	: **CBCS (LOCF) B.Sc. (H) Mathematics**
Unique Paper Code	: **32351502**
Name of Paper	: **BMATH-512 GROUP THEORY-II**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Show that the automorphism groups of two isomorphic groups are isomorphic. Let G be a group and $g \in G$ be an element of finite order. Show that $|\phi_g|$ divides $|g|$, where ϕ_g is the inner automorphism of G generated by g. Give an example of a group G and an element $g \in G$ for which $1 < |\phi_g| < |g|$.

2. Determine the number of elements of order 15 in the group $\mathbb{Z}_{75} \oplus \mathbb{Z}_{20}$. Also determine the number of cyclic subgroups of order 15 in this group. Is the group $\mathbb{Z}_{75} \oplus \mathbb{Z}_{20}$ isomorphic to the group $\mathbb{Z}_{25} \oplus \mathbb{Z}_{60}$? Justify your answer.

3. Find all Abelian groups (up to isomorphism) of order 100. From these isomorphism classes determine those classes that have elements of order 25. Does every Abelian group of order 100 has a cyclic subgroup of order 10? Justify your answer.

4. State the following statements as True or False. Justify your answer with proper reasoning.
    a. The action of D_{20} (the dihedral group of order 20) on itself by conjugation is faithful.
    b. Let S be a finite set and G be a subgroup of $Sym\ (S)$. Let $\sigma \in G$ and $s \in S$. If G acts transitively on S, then $\bigcap_{\sigma \in G} \sigma\, G_s\, \sigma^{-1} = \{I\}$, where G_s denotes the stabilizer of s in G.
    c. A group of order 160 is not simple.
    d. If two groups have same class equation, then the groups are isomorphic.
    e. Any two 3-cycles in A_4 are conjugate.

5. Let $G = D_{10}$ (the dihedral group of order 10) and $A = \{1, r, r^2, r^3, r^4\}$, where r denotes the rotation of regular pentagon by 72° about the centre in clockwise direction. Find $C_G(A)$ and $N_G(A)$. Let H be a subgroup of order 2 in G. Show that $N_G(H) = C_G(H)$. Deduce that if $N_G(H) = G$, then H is a subgroup of $Z(G)$. Let $G = D_8$ and G acts on itself by the left regular action. By labelling the elements $1, r, s, sr, r^2, sr^2, r^3, sr^3$ of G with the natural numbers $1, 3, 5, 7, 2, 4, 6, 8$ respectively, where r denotes the rotation of a square by 90° in clockwise direction and s denotes the reflection of square about the line passing through the vertices 1 and 3, exhibit the image of each element of G under left regular representation of G into S_8.

6. Find Sylow subgroups of a group G of order 108. Show that either G has a normal Sylow 3-subgroup or G contains a normal subgroup of order 9.