Name of the Course	**: B.Sc. (H) Mathematics (CBCS)**
Unique Paper Code	**: 32357505_OC**
Name of the Paper	**: DSE- Discrete Mathematics**
Semester	**: V Semester**
Duration	**: 3 hours**
Maximum Marks	**: 75 Marks**

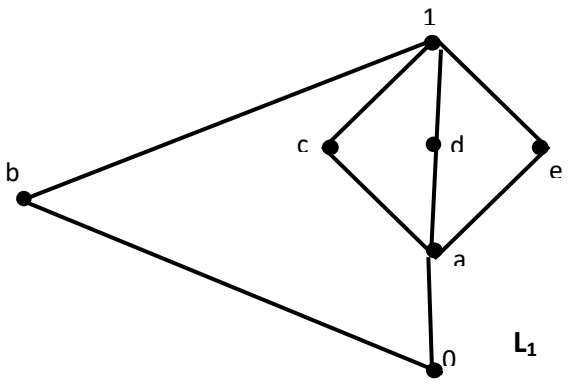
Instructions for Candidates:

Attempt any four questions. All questions carry equal marks.

1. Let P $= \{1, 2, 3, 5, 6, 10, 15, 30\}$. Show that P is a partial ordered set with divisibility as partial order relation. Draw Hasse Diagram of *P*. Find lower bounds and upper bounds of 10 and 15. Draw the diagrams for the ordered sets $\mathbf{2} \cup \mathbf{3}$, $\mathbf{2} \times \mathbf{3}$, and $\mathbf{2} \oplus \mathbf{3}$ where **n** denotes the chain $\mathbf{0} < \mathbf{1} < \mathbf{2} < \mathbf{3}$ ….…..$< \boldsymbol{n} - \mathbf{1}$.

2. Given lattice $L = \{1,3,5,7,15,21,35,105\}$ of positive divisors of 105 with respect to the given order $x \leq y \iff x$ divides *y*, Find sublattices *S* and *T* of L such that the union, $S \cup T$ is not a sublattice of *L*,but $S \cap T$ is a lattice. Prove or disprove that every chain is a lattice. Is every lattice also a chain? Justify your answer by verifying the same for the lattice *L* defined above. Prove or disprove that lattice $(P(X), \cap, \cup)$, where $X = \{1, 2\}$ is isomorphic to the sublattice $R = \{1,3,15,105\}$ of L. Hence or otherwise, find a sublattice of *L* that is isomorphic to $(P(X), \cap, \cup)$. Justify your answer.

3. Verify whether the lattice L_1 in the figure given below is modular and /or distributive, by using M_3-N_5 theorem.



Find elements a, b, c in M_3 which do not satisfy the following law:

(a ∨ b = c ∨ b and a ∧ b = c ∧ b) => a= c

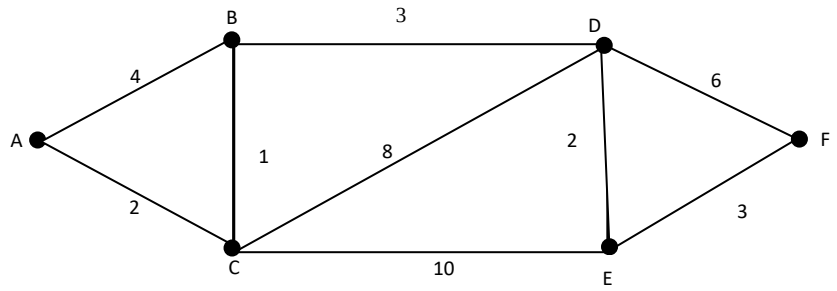
Also prove that in a distributive lattice L the above law holds for all a,b,c ∈ L.

4. Find the disjunctive normal form of $p = x'y + xy'z + xy'z' + x'y'z$ in three variables and simplify it using Karnaugh Map or Quine-McCluskey method. Give the symbolic representation of the circuit *q* given by

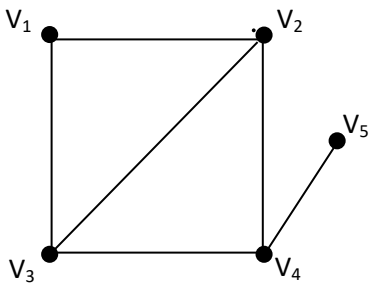
$$q = x'yz((x + z') + (y' + z'))$$

Also,draw the contact diagram of above circuit q.

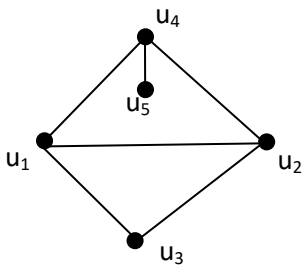
5. Use the improved version of Dijkstra's Algorithm to find the length of shortest path from A to F in the graph given below. Show the final labels on all vertices, Explain Algorithm.



Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find an isomorphism between the graph G_1 and G_2.



G_1



G_2

6. Does there exist a graph with 28 edges and 10 vertices, each of degree 4 or 6? Justify your answer. Does there exist the graphs whose degree sequence are (a) 4,3,2,2,1 (b) 5,4,3,2,1 ? In each case either draw the graph or explain why no graph exists.

For pair of graphs shown below, either label the graphs so as to exhibit an isomorphism or explain why graphs are not isomorphic.