Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351601**
Name of Paper	: **C 13- Complex Analysis**
Semester	: **VI**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Determine whether $S = \{z \in \mathbb{C} : |z|^2 > z + \overline{z}\}$ is a domain or not? Justify your answer.

Find the image of line segment joining $z_1 = -i$ to $z_2 = -1$ under the map $f(z) = \overline{iz}$.

Check whether Cauchy-Riemann equations for $f(z) = \sqrt{|z^2 - \overline{z}^2|}$ are satisfied at the origin? Is f analytic at the origin? Justify your answer.

Suppose $f(z) = \cosh(2x)\cos(2y) + i\, v(x,y)$ is analytic everywhere such that $v(0,0) = 0$. Find $f(z)$. Hence find zeros of f.

Solve the equation $e^{z-1} + ie^3 = 0$.

2. Let $S = \{z \in \mathbb{C} : \text{Im } z = 1 \text{ and Re } z \neq 4\}$. Is S open? Is S closed? Justify your answer.

Assume that g is analytic in a region and that at every point either $g = 0$ or $g' = 0$. Show that g is constant.

Suppose $f(z) = \begin{cases} \overline{z}^3/z^2 & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$. Show that f is continuous everywhere on $\mathbb{C}$. Is f analytic at $z = 0$? Justify your answer.

Does there exists an analytic function $f(z) = u(x,y) + i\, v(x,y)$ for which $u(x,y) = y^3 + 5x$? Solve the equation $Log(z) + Log(2z) = 3\pi/2$.

3. Determine whether the following curves are simple, closed, smooth or contour

C_1: $z(t) = |t| + it$, $t \in [-1,1]$

C_2: $z(t) = e^{2it}$, $t \in [0, 2\pi]$,

C_3: $z(t)$ is the positively oriented boundary of the rectangle whose sides lie along $x = \pm 1,\ y = 0,\ y = 1$.

Evaluate $\int_{C_3} |z| dz$. Explain why Cauchy Goursat theorem is not applicable in this case?

Use ML-Inequality to show that

$$\left|\int_C \frac{e^z}{(z+1)} dz\right| \leq 4\pi e^2$$

where C : $z(t) = e^{2it}$, $t \in [-\pi, \pi]$.

4. Evaluate $\int_C ze^{3z} dz$ where C is the parabola $x^2 = y$ from $(0,0)\ to\ (1,1)$.

Using Cauchy Integral formula, determine the integral $\int_C \frac{e^z}{z^2(z^2-9)} dz$ where C is positively oriented circle (i) C: $|z| = 1$. (ii) C: $|z-3| = 1$.

Use Liouville's theorem to establish that $cos\ z$ is not bounded in the complex plane.

Let g be an entire function and suppose that $|g(z)| < 10$ for all values of z on the circle $|z-2| = 3$. Find a bound for $|g'''(2)|$.

5. Determine the radius of convergence of the series $\sum_{k=0}^{\infty} \frac{z^k}{k!}$ and $\sum_{k=0}^{\infty} k^k z^k$. Also discuss the convergence of the series.

Obtain the Maclaurin series of the function $(z) = \frac{1}{z^2} sinh\left(\frac{1}{z}\right)$. Specify the region in which the series is valid.

Find the Laurent series of the function $f(z) = \frac{1}{(z+1)(z+3)}$ valid for $0 < |z+1| < 2$.

6. Determine the residue and singularities of the function $g(z) = \frac{z+1}{z^2+4}$. Also evaluate $\int_C g(z)dz$ where C is the positively oriented circle $|z-i| = 2$.

Using a single residue, evaluate the integral $\int_{C'} \frac{3z-1}{z(z+1)} dz$ where C' is the positively oriented circle $|z-1| = 4$.

Use residue to evaluate the integral $\int_0^{2\pi} \frac{dt}{3+cost}$