Name of Course	: **CBCS B.Sc. (H) Mathematics**
Unique Paper Code	: **32351101**
Name of Paper	: **BMATH101-Calculus**
Semester	: **I**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Sketch the graph of the function

$$f(x) = 3x^4 - 4x^3$$

by finding the intercepts, critical numbers, intervals of increase/decrease, relative extrema, second-order critical numbers, concavity and inflection points.

It is projected that t years from now, the population of a certain country will be

$$P(t) = 50\, e^{0.02t} \text{ million.}$$

(i) At what rate the population is changing with respect to time 10 years from now?
(ii) At what percentage rate will the population be changing with respect to time t years from now?

Find the nth order differential coefficient of

$$y = \sin x \log(ax + b).$$

(7.75 + 6 + 5)

2. Convert the polar equation $r = 4\cos\theta + 6\sin\theta$ to rectangular coordinates. Show that it represents a circle. Find the centre and radius of that circle.

Identify and sketch the following conic by removing the xy-term

$$8x^2 - 12xy + 17y^2 = 20.$$

Find the equation of hyperbola with vertices $(0, \pm 3)$ and asymptotes $y = \pm x$.

(6 + 8.75 + 4)

3. Let R be the region bounded in the first quadrant by the curves $y = x^2$, the y-axis and the line $y = 1$. Determine the volume of the solid generated when R is revolved about the line $x = 2$ using cylindrical shell method and washer method.

Find the area of surface generated by the revolving the curves

(i) $y = \sqrt{4 - x^2}$, $-1 \le x \le 1$ about x-axis,
(ii) $x = y^3$, $0 \le y \le 1$ about the y-axis.

(10.75 + 8)

4. Find the vector limit

$$\lim_{t\to 0+}\left[\left(1+\frac{1}{t}\right)^{t}\boldsymbol{i}-\left(\frac{\sin t}{t}\right)\boldsymbol{j}-\left(\frac{e^{-t}}{1-t}\right)\boldsymbol{k}\right].$$

A projectile is fired from ground level with muzzle speed 50 ft/s at an angle of elevation of $\alpha = 30^0$. What is the maximum height reached by the projectile? What is the time of flight and the range?

A particle moves along a path given in parametric form where $r(t) = 3 + 2\sin t$ and $\theta(t) = t^3$. Find the velocity and acceleration of the particle in terms of the unit polar vectors $\boldsymbol{u_r}$ and $\boldsymbol{u_\theta}$.

Find unit tangent $\boldsymbol{T}(t)$ and unit normal $\boldsymbol{N}(t)$ of the curve given by $r(t) = (t^2+1)\,\boldsymbol{i} + t\,\boldsymbol{j}$ at $t = 1$.

(3.75 + 5 + 5 + 5)

5. Let

$$L = \lim_{x\to\pi/2}(\sin x)^{\tan^2 x}$$

and

$$M = \lim_{x\to(\pi/2)-}(\tan x)^{\sin(2x)}.$$

Find the values of L and M and show that $eL^2 = M$.

Prove that $\cosh^{-1}x = \ln\left(x + \sqrt{x^2-1}\,\right), x \geq 1.$

Find the centre, vertices, foci and ends of minor axis of the ellipse

$$3x^2 + 4y^2 - 30x - 8y + 67 = 0.$$

(7.75 + 6 + 5)

6. If $y = \log(x + \sqrt{x^2+1})$, prove that

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2y_n = 0.$$

The position of an object moving in space is given by

$$R(t) = (e^{-t}\cos t)\boldsymbol{i} + (e^{-t}\sin t)\boldsymbol{j} + e^{-t}\boldsymbol{k}.$$

Find the velocity, speed and acceleration of the object at arbitrary time t and at $t = 0$. Also, determine the curvature of the trajectory at arbitrary time t and at $t = 0$.

Prove that

$$\int x^n e^x dx = x^n e^x - n\int x^{n-1}e^x dx.$$

Hence, evaluate $\int x^2 e^{3x} dx$.

(6 + 6 + 6.75)