(d) Given $\frac{dy}{dt} = 4e^{0.8t} - 0.5y$, $0.5 < t < 1.5$, $h = 0.5$, $y(0) = 2$. Find approximate solution using Heun's method. (6.25)

6. (a) Calculate $\int_1^2 \frac{dx}{x^2}$ using Richardson extrapolation. (6.25)

(b) Evaluate $\int_1^2 e^x \, dx$ using Simpson's rule taking $h = \frac{1}{2}$. Find a bound on the error and compare it with the exact solution. (6.25)

(c) For the given data, find $f''(1.05)$: (6.25)

x	1.00	1.01	1.02	1.03	1.04	1.05
f(x)	1.27	1.32	1.38	1.41	1.47	1.52

(d) Use Euler's method with step-size h = 0.3 to compute the approximate y-value y(0.9) of the solution of the initial value problem (6.25)

$$\frac{dy}{dx} = x^2, \quad y(0) = 1.$$

[This question paper contains 6 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 2597 **A**

Unique Paper Code : 32355402-OC

Name of the Paper : GE-4 Numerical Methods

Name of the Course : **Generic Elective CBCS (Other than Maths(H.))**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Use of Scientific calculator is allowed.
3. **All** questions are compulsory. Attempt any **two** parts from each question.
4. Each question carries equal marks.

1. (a) Perform three iterations of the Bisection method to find a root of the equation f(x) = 1.05 − 1.04x + lnx = 0. Also represent the root graphically. (6.25)

(b) Write out the Newton form of the interpolating polynomial for $f(x) = e^x$ that passes through the points $(-1, e^{-1})$, $(0, e^0)$, $(1, e^1)$. Also, evaluate f(0.5) and find the approximate error. (6.25)

(c) Find a root of the equation $f(x) = \sin x - (2 + x^3) = 0$ by applying Newton Raphson method. Start with $x(0) = -1.25$ as the initial approximation and perform three iterations. (6.25)

(d) Perform three iterations of Regula-Falsi method to find the root of the equation $\cos x - x = 0$ in the interval (0,1). (6.25)

2. (a) Define rate of convergence of an iterative method. Determine the rate of convergence of the Secant method. (6.25)

(b) Find the smallest positive root of the equation $x - e^{-x} = 0$ using the Secant method. Perform three iterations. (6.25)

(c) Construct the interpolating polynomial that fits the data using the Gregory-Newton Forward Difference interpolation. Hence evaluate the values of f(x) at x = 0.15 and 0.25. (6.25)

x	0	0.1	0.2	0.3
f(x)	–1.5	–1.27	–0.98	–0.64

(d) Using Newton's Forward difference formula, find the polynomial f(x) satisfying the following data:

x	0	1	2	3
f(x)	1	2	1	10

Hence find f(0.5). (6.25)

5. (a) Find $\int_0^5 \frac{dx}{1+x^2}$ using trapezoidal rule taking h = 1. (6.25)

(b) Use Romberg's method to compute $\int_4^{5.2} \log x \, dx$ taking the data : (6.25)

x	4	4.2	4.4	4.6	4.8	5.0	5.2
$\log_e x$	1.3863	1.4351	1.4816	1.526	1.5686	1.6094	1.6486

(c) Obtain the piecewise linear interpolating polynomial for the function f(x) defined by the data :

x	0	2/3	1	2
f(x)	4	–4	–7/2	–1/2

Hence estimate the values of f(1/2) and f(3/2). (6.25)

(d) Use the formula $f'(x)=\frac{f(x)-f(x-h)}{h}$ to approximate the derivative of f(x) = sinx at x = π, taking h = 1,0.1,0.01 and 0.001. (6.25)

3. (a) Find the inverse of the following matrix using the Gauss-Jordan method : (6.25)

$$\begin{bmatrix} 3 & -1 & 2 \\ 1 & 1 & 2 \\ 2 & -2 & -1 \end{bmatrix}$$

(b) Use finite difference method to solve the Boundary value problem

$$y''(x) = 4(y - x),\ 0 \le x \le 1,$$

with y(0) = 0, y(1) = 2 and h = 0.4. (6.25)

(c) Perform three iterations of Gauss-Seidal method for the following system of equations:

$$2x - y = 7,$$

$$-x + 2y - z = 1,$$

$$-y + 2z - 1,$$

using initial approximation (0, 0, 0). (6.25)

(d) Approximate the following integral using Gaussian quadrature method with $n = 2$ $\int_1^{1.5} x^2 \ln(x)\, dx$.

(6.25)

4. (a) Solve the following system of equations using Gauss-Jacobi method:

$$4x + y + z = 2,$$

$$x + 2y + 3z = -4,$$

$$x + 5y + 2z = -6.$$

Perform three iterations using initial approximation (0.5, 0.5, −0.5). (6.25)

(b) Solve the following system of equations by Gauss Elimination method: (6.25)

$$x + 2y + 3z = 1,$$

$$x + 3y + 5z = 2,$$

$$2x + 5y + 9z = 3.$$

(c) Show that : (6.25)

(i) $(1+\Delta)(1-\nabla)=1$,

(ii) $\mu\delta = \dfrac{\Delta + \nabla}{2}$.