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| Name of the Course | : **Generic Elective** |
| Unique Paper Code | : **32355301** |
| Name of the Paper | : **GE-3 Differential Equations** |
| Semester | : **III** |
| Duration | : **3 Hours** |
| Maximum Marks | : **75 Marks** |

*Attempt any four questions. All questions carry equal marks.*

**1.** (i) Solve

$$xdy - ydx = \sqrt{x^2 + y^2}\,dx.$$

(ii) Solve the initial value problem

$$\left(x^2 + y^2 + x\right)dx + xy\,dy = 0, \qquad y(1) = 1.$$

(iii) Solve the initial value problem

$$x\frac{dy}{dx} + y = y^2 \log x, \qquad y(1) = -1.$$

**2.** (i) Find the orthogonal trajectories of the family of curves $3xy = x^3 - a^3$, $a$ being parameter of the family.

(ii) Find a family of oblique trajectories that intersect the family of circles $x^2 + y^2 = c^2$ at angle $45^\circ$.

(iii) Solve

$$\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}.$$

**3.** (i) Solve by method of variation of parameters

$$y'' + y = \operatorname{cosec} x.$$

(ii) Solve by method of undetermined coefficients

$$y'' + 1.44y = 24\cos 1.2\ x.$$

(iii) Solve

$$y''' - 2y'' + 4y' - 8y = 0,\ y(0) = -1,\ y'(0) = 30,\ y''(0) = 28.$$

**4.** (i) Show that $\{e^{-x}, e^{3x}, e^{4x}\}$ forms a basis of the solution set of the equation

$$\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 12y = 0.$$

(ii) Solve the initial value problem

$$x^2 y'' - 2x y' - 10y = 0, \quad y(1) = 5, \quad y'(1) = 4.$$

(iii) Solve the linear system

$$y_1' = 2y_1 + 5y_2$$
$$y_2' = 5y_1 + 12.5y_2$$

**5.** (i) Find the partial differential equation arising from the surface

$$z = xy + f\left(x^2 + y^2\right).$$

(ii) Find the general solution of the partial differential equation

$$u_x + 2xy^2 u_y = 0.$$

(iii) Apply the method of separation of variables $u(x, y) = f(x)g(y)$ to solve

$$y u_x + x u_y = 0 \quad \text{on} \quad u(0, y) = y^2.$$

**6.** Reduce each of the following equations into canonical form and find the general solution:

(i) $u_x - u_y = u, \quad u(x, 0) = 4e^{-3x}$.

(ii) $u_{xx} + 6u_{xy} + 9u_{yy} + 3y u_y = 0.$

(iii) $u_{xx} - 3u_{xy} + 2u_{yy} = 0.$