[This question paper contains 3 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 1849 **A**

Unique Paper Code : 32375201

Name of the Paper : Introductory Probability (GE-II)

Name of the Course : B.Sc. (H) **Statistics under CBCS**

Semester : II

Duration : 3½ Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **Section A** is compulsory.
3. Attempt any **five** questions, selecting at least **two** questions from each of the **Sections B** and **C**.
4. Use of simple calculators is allowed.

Section A

1. Answer the following:

(a) Let $f(x) = c\left(\frac{1}{4}\right)^x$ for x = 1, 2, 3,........ then the value of c for which $f(x)$ can serve as a probability mass function is---------------. [1]

(b) Define simple and composite events with examples. [1]

(c) X and Y are independent events and E(X) = 10, E(Y) = 20, then value of E(XY) is? [1]

(d) Name the distribution which exhibits lack of memory. [1]

(e) If $M_X(t) = \left(0.9 + (0.1)\,e^t\right)^{100}$, then value of $E\left(X^2\right)$ is ? [1]

(f) If $X \sim P(4)$ *and* $Y = X - 4$, *then* $M_{Y(t)} =$ --------------. [2]

(g) X and Y are independent normal variates with means 1, 2 and variances 25 and 36 respectively. Compute $E\left(e^{\alpha X + \beta Y}\right)$. [2]

(h) If $E(X) = \mu$ and $Var(X) = \sigma^2$ and using Chebychev's inequality we get $E(|X - \mu \le k\sigma|) > 1 - a$, then value of a is ? [2]

(i) If $\mu_X = 10$, $\mu_Y = 20$, $\sigma_X^2 = 25$, $\sigma_Y^2 = 36$ and $Cov(X, Y) = 100$, then what is the value of Var (2X - 3Y)? [2]

(j) State De moivre Laplace central limit theorem. [2]

Section B

2. (a) A box contains 20 fuses, of which five are defective. If three of the fuses are selected at random and are removed from the box in succession, what is the probability that all are defective?

(b) Let X be a discrete random variable having the probability density function:

$$f(x) = \frac{|x-2|}{7} \quad for\ x = -1, 0, 1, 3$$

Obtain the moment generating function of X. Hence obtain the mean and variance of X. [6,6]

3. (a) Two socks are selected at random and are removed in succession from a drawer containing five brown socks and three green socks. If random variable W represents the number of brown socks selected then find the probability distribution of W and E(W).

(b) The amount of bread (in hundreds of pounds) x that a certain bakery is able to sell in a day is found to be a random phenomenon with a probability density function given by:

$$f(x) = \begin{cases} kx, & 0 \le x < 5 \\ k(10 - x), & 5 \le x < 10 \\ 0, & otherwise \end{cases}$$

i. Find the value of k such that f(x) is a probability density function.
ii. What is the probability that the number of pounds of bread that will be sold tomorrow is: (i) more than 500 pounds (ii) less than 500 pounds and (iii) between 250 and 750 pounds?

[6,6]

4. (a) Two percent of books bound at a certain bindery have defective bindings. Use Poisson distribution to determine the probability that 5 of 400 books bound by this bindery will have defective bindings.

(b) Let $X \sim \exp(4)$. Then, find $P(X \ge 5)$ and $P(4 \le X \le 7)$.

[6,6]

5. (a) Three random variables X, Y and Z have means $\mu_X = 2$, $\mu_Y = -3$, $\mu_Z = 4$, variances $\sigma_X^2 = 1$, $\sigma_Y^2 = 5$, $\sigma_Z^2 = 2$ and $Cov(X, Y) = -2$, $Cov(Y, Z) = -1$ and $Cov(Z, X) = 1$. Find Cov (U, V) where U = 3X - Y + 2Z and V = X - 3Y + Z.

(b) Find mean and variance of random variable X whose probability density function is:

$$f(x) = \begin{cases} \dfrac{4}{\pi\left(1 + x^2\right)}, & 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

[6,6]

Section C

6. (a) An item is produced in large numbers. The machine is known to produce 5% defectives. A quality control inspector is examining the items at random. What is the probability that at least 4 items are to be examined in order to get 2 defectives?

(b) If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.75, what is the probability that an applicant will finally pass the test on the fourth try?

[6,6]

7. (a) X has discrete uniform distribution, f(x) = 1 / k, for x = 1, 2, ..., k . Show that the moment generating function is given by

$$M_X(t) = E\left(e^{tX}\right) = \frac{e^t\left(1 - e^{kt}\right)}{k\left(1 - e^t\right)}$$

. Also find $M_X'(0)$, $M_X''(0)$ and hence compute mean and variance of random variable X.

(b) Let X_1 and X_2 be two independent Poisson variates with parameters λ_1 *and* λ_2 respectively. Prove that $X_1 + X_2$ follows Poisson distribution with parameter $\lambda_1 + \lambda_2$. Does $X_1 - X_2$ also follows Poisson distribution? Give reasons.

[6,6]

8. (a) Define Beta distribution of first kind and find the mean and variance.

(b) If X is binomially distributed with parameters n and p, then what is the distribution of Y = n - X?

[6,6]

9. (a) Subway trains on a certain line run every half hour between midnight and six in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least twenty minutes?

(b) Obtain moment generating function of the Geometric distribution. Also, find its mean and variance. [6,6]