What is the worst-case running time of this algorithm? Explain your answer. Can the running time be reduced? Justify your claim. (6)

18/5/22

[This question paper contains 10 printed pages.]

Your Roll No.....

Sr. No. of Question Paper : 2134 **A**

Unique Paper Code : 32341401 OC

Name of the Paper : Design and Analysis of Algorithms

Name of the Course : **B.Sc. (Hons.) Computer Science**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. **1** of **35** marks is compulsory.
3. Attempt any **Four** questions from Q. No. **2** to Q. No. **8.**

1. (a) A teacher organized a quiz competition in a class of n students. The teacher wishes to announce the top and the least score in the class. Suggest an optimal algorithm that can be used to find the two scores in less than $2n-2$ comparisons. Also,

analyse and explain the time complexity of the suggested algorithm. (4)

(b) Give the recurrence for the best-case running time of Quick Sort algorithm and compute the time complexity using the same. (4)

(c) Consider an instance of the Subset Sum Problem where bound W=6 and three items with weights w1 = 1, w2 = 4, w3 = 3. With the help of this example, illustrate that the memorized recursive algorithm solves lesser number of subproblems than the corresponding iterative algorithm. (4)

(d) Illustrate the step-by-step execution of the Bucket Sort algorithm on the given array A.

A = <0.3, 0.2, 0.5, 0.4, 0.9, 0.67> (4)

(e) Suppose we perform a sequence of stack operations (push()/pop()) on a stack whose size never exceeds *k*. After every *k* operations a copy operation is performed, wherein a copy of the entire stack is made for backup purpose. Show that the cost of *n* stack operations *(n>=k)*, including the periodic copying of the stack, is *O(n)*. (4)

8. (a) Suppose that there are *n* numbers between *1* and *50* in a Red Black tree and number *18* is needed to be searched. For each of the following sequences in which the nodes are examined, state whether the given order is possible or not. Justify the answer.

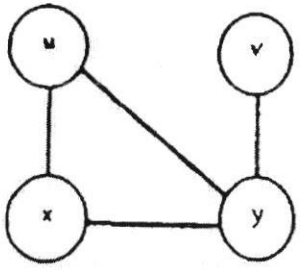
(i) 27, 29, 14, 16, 18

(ii) 36, 13, 27, 17, 18 (4)

(b) Consider the following recursive algorithm to find an optimal schedule for the Weighted Interval Scheduling problem, where the intervals are labelled from *1, ... n,* having a start time *Si* and a finish time f_i. Each interval *i* has a value, or weight v_i, and the intervals are sorted in the order of non-decreasing finish time. Further, *P(j)* for an interval *j*, is defined as the largest index $i < j$ such that intervals *i* and *j* are disjoint.

```
Compute-Opt (int j)
  if j = 0 then
        return 0
  else
        return max (v_j + Compute-Opt (P(j)),
        Compute-Opt (j-1))
```

(b) Give an algorithm to find whether a graph is bipartite or not For the graph given below, specify whether the graph is bipartite or not. Justify the answer.



(5)

7. (a) Consider an adjacency list representation of a directed graph wherein an array of the outgoing edges (and not the incoming edges) for each vertex is maintained. Give an algorithm to compute the total number of incoming edges for a given vertex and discuss the time and space complexity of the algorithm. (6)

(b) Is it always true that an array that is already sorted in ascending order is a best-case input for sorting algorithms? If yes, explain the reason. If no, state an algorithm for which it is not a best-case input and explain your answer. (4)

(f) Let L_0, L_1, L_2, ... be the layers obtained by performing breadth first search on a given graph G = (V, E), where V is the set of vertices and E is the set of edges. Further, let e = (u,v) be an edge in E such that $u \in L_i$ and $v \in L_j$, is it possible that $|i-j| \geq 2$? Justify your answer. (3)

(g) Explain a scenario in which the naive string-matching algorithm demonstrates its worst-case behaviour. (3)

(h) For the Interval Scheduling Problem, does the greedy strategy of selecting the available request that starts earliest (the one with minimal start time s(i)) provide an optimal solution to the problem? If yes, justify the answer. If no, give a counter example. (3)

(i) Give the running time of search operation on a Binary Search Tree and Red Black Tree. For which of the two, is the search operation more efficient and why? (3)

(j) Compare the space requirements of adjacency list and adjacency matrix representations of a graph having *m* edges and *n* vertices. (3)

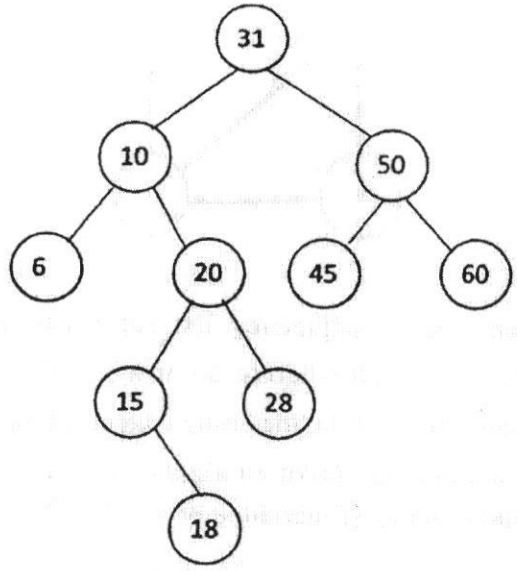
2. (a) What does the following algorithm compute?

```
MyFunc (int a [], int n)
{
    if (n = = 1)
            return
    else
    {
            MyFunc (a, n-1)
            k = a[n]
            j = n-1
            while (j>=1 && a[j] > k)
            {
                a[j+1] = a[j]
                j--
            }
            a[j+1] = k
            return
    }
}
```

State and solve the recurrences for the best-case and worst-case running time of this algorithm and compute the time complexity in asymptotic notation. (5)

(b) Consider an Operating System that receives job requests along with their start time and burst time.

(b) Color the nodes of the following binary search tree with red or black colors so that the resulting tree is a Red-Black tree. Delete the keys 10, 15, 50 successively from the obtained tree.



(6)

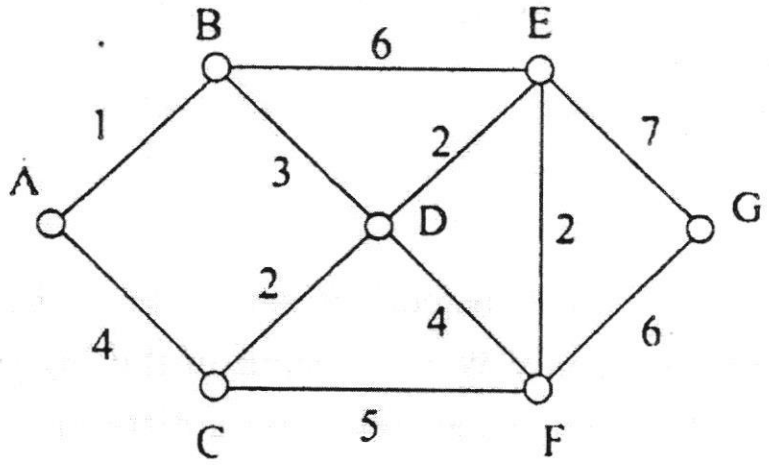
6. (a) What is the worst-case running time of Randomized-Select algorithm. Suppose we use this algorithm to select the maximum element of the array A = <13, 25, 39, 0, 67, 85>. What sequence of the chosen pivots will result in a worst-case performance? Justify your answer. (5)

Give an efficient algorithm for the child to fill the bags with sand so that the number of bags used is minimized. (3)

(b) Describe an algorithm that, given n integers in the range 0 to k, preprocesses its input and then answers any query about how many of the n integers fall into a range [a ... b] in O(1) time. Your algorithm should use $\theta(n+k)$ preprocessing time. (3)

(c) Give an algorithm to print the first n numbers of the Fibonacci series using memorized recursion method. State the running time of the algorithm. (4)

5. (a) Find the minimum spanning tree of the following graph using Kruskal's algorithm.



(4)

A job request may or may not be executed. If executed, the job requests are executed beginning at their start time. The Operating System receives all the job requests in advance. Also, the underlying hardware is a single core machine that can execute only one job request at a time. So, only those job requests that do not overlap with each other can be executed sequentially. Write a scheduling algorithm for this Operating System that should optimize the number of job requests executed. (5)

3. (a) Can we use Depth First Search algorithm to find the shortest-path distance from the source to a reachable vertex in an unweighted graph? If yes, justify. If no, give a counter example. Would you use Breadth First Search to find the shortest path between two nodes in a weighted graph with arbitrary edge weights? Why or why not? (6)

(b) Sort the following character array using Heapsort algorithm

T R Y H A R D (4)

4. (a) A child wants to collect sand from a beach, but can carry only W kg of sand in the luggage. The child has n bags each having a different capacity.