Name of Course : **CBCS (LOCF) B.A. (Prog.)**
Unique Paper Code : **62351101**
Name of Paper : **Calculus**
Semester : **I**
Duration : **3 hours**
Maximum Marks : **75 Marks**
Attempt any four questions. All questions carry equal marks.

1. Test the continuity and differentiability of the following function at $x = 0$ and $x = \pi/2$

$$f(x) = \begin{cases} 1 & -\infty < x < 0 \\ 1 + \sin x & 0 \le x < \pi/2 \\ 2 + \left(x - \frac{\pi}{2}\right)^2 & \pi/2 \le x < \infty \end{cases}$$

Also, find the points at which the function

$$g(x) = |x + 1| + |x - 2|$$

is not differentiable.

2. If $y = (\sin^{-1} x)^2$, prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - n^2 y_n = 0$$

and find $y_n(0)$. If $u = \tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin u\,.$$

3. Find the radius of curvature of the following curves:

(i) $\sqrt{x} + \sqrt{y} = 1$ at $(1/4\,,1/4)$

(ii) $y = e^x$ at the point where it meets y-axis.

Determine the nature and position of the double points for the following curves:

(i) $x^4 - 4y^3 - 12y^2 - 8x^2 + 16 = 0$

(ii) $y^2 = bx\sin(x/a)$

Prove that ρ^2/r is a constant for the cardioid $r = a(1 + \cos\theta)$, where ρ denotes the radius of curvature.

4. Find the equations of the tangent and normal at any point of the curve

$$x = ae^{\theta}(\sin\theta - \cos\theta), \quad y = ae^{\theta}(\sin\theta + \cos\theta)\,.$$

Find the asymptotes of the following curve

$$x^3 + x^2y - xy^2 - y^3 + x^2 - y^2 - 2 = 0\,.$$

Also, trace the curve

$$(a^2 + x^2)y = a^2x\,.$$

5. Verify Rolle's theorem for the function given by

$$f(x) = x^3 - 6x^2 + 11x - 6 \quad in \quad [1,3]$$

Use Lagrange's Mean Value Theorem to prove that

$$1 + x < e^x < 1 + xe^x, \quad where \quad x > 0\,.$$

Also, show that

$$\frac{\sin\alpha - \sin\beta}{\cos\beta - \cos\alpha} = \cot\theta \ , \ where\ 0 < \alpha < \theta < \beta < \frac{\pi}{2}.$$

6. Find by Maclaurin's Theorem, the first four terms and the remainder after n terms of the expansion of $e^{ax}\cos bx$ in a series of ascending powers of x.
   Determine $\lim_{x\to 0}(\cot x)^{1/\log x}$ and $\lim_{x\to 0}\left(\frac{1}{x^2} - \cot^2 x\right)$. Further, show that $f(x) = \sin x\,(1+\cos x)$ is maximum when $x = \pi/3$.