(b) Test for convergence the series whose nth term is $\frac{n^{n^2}}{(n+1)^{n^2}}$.

(c) Test for convergence and absolute convergence of the following series :

$$1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\dots$$

6. (a) Show that the function f defined by

$$f(x)=\begin{cases}0, & \text{when x is rational}\\ 1, & \text{when x is irrational}\end{cases}$$

is not integrable on any interval.

(b) Show that every Monotonic function on [a, b] is integrable on [a, b]

(c) Show that the function f(x) = [x], where [x] denotes the greatest integer not greater than x, is integrable over [0, 3] and $\int_0^3 [x]dx = 3$.

[This question paper contains 4 printed pages.]

Your Roll No...............

Sr. No. of Question Paper : 522 **B**

Unique Paper Code : 62354443

Name of the Paper : Analysis (CBCS) (LOCF)

Name of the Course : **B.A. (Prog.)**

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.
3. Attempt any **two** parts from each question.
4. **All** questions carry equal marks.

1. (a) State the Completeness Property of $\mathbb{R}$. Show that

$$\inf S = 2 \text{ and } \sup S = \frac{1}{2}, \text{ where } S=\left\{1-\frac{(-1)^n}{n} : n\in\mathbb{N}\right\}.$$

(b) Let S be a non-empty subset of $\mathbb{R}$ that is bounded above and let a be any number in $\mathbb{R}$. Prove that $\sup(a+S) = a + \sup S$.

(c) Determine the points of continuity of greatest integer function $f(x) = [x]$, $x \in \mathbb{R}$.

2. (a) Show that $f(x) = 1/x$ is continuous on $\mathbb{R} \sim \{0\}$.

(b) Let $f: R \to R$ be defined as

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 0, & \text{if } x \text{ is irrational} \end{cases}$$

Show that f is not continuous at any point of $\mathbb{R}$.

(c) Show that $f(x) = x^2$ is uniformly continuous on $[-2, 2]$.

3. (a) State Cauchy's general principle of convergence.

Apply it to prove that the sequence $\langle a_n \rangle$ defined by

$$a_n = 1 + \frac{1}{4} + \frac{1}{7} + \dots + \frac{1}{3n-2} \text{ is not convergent.}$$

(b) Prove that a sequence of real numbers converges if and only if it is a Cauchy sequence.

(c) A sequence $\langle a_n \rangle$ is defined as follows :

$$a_1 = 1,\ a_{n+1} = \frac{4+3a_n}{3+2a_n},\ n \geq 1$$

Show that sequence $\langle a_n \rangle$ converges and find its limit.

4. (a) Define the sequence of partial sums of a series. Using the sequence of partial sums, test the convergence of the following series :

$$\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \cdots$$

(b) Show that the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \tan\frac{1}{n}$ is convergent.

(c) Test the series

$$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots$$

for convergence for all positive values of x.

5 (a) Test the convergence of the following series:

$$\sum_{n=1}^{\infty} \sqrt{n^4+1} - \sqrt{n^4-1}\ .$$