(b) Let $G=\left\{\begin{bmatrix} a & a \\ a & a \end{bmatrix} : a \in \mathbb{R},\ a \neq 0\right\}$. Show that G is a group under matrix multiplication.

(c) If $\sigma=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}$ and $\rho=\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix}$ are two permutations, Compute the values of $\sigma^{-1}\rho\sigma$ and $\rho^2\sigma$.

6. (a) Prove that the set of all matrices of the form $\left\{\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{Z}\right\}$ is a subring of the ring of all 2×2 matrices over $\mathbb{Z}$.

(b) If A & B are subrings of a ring R. Then $A \cap B$ is also a subring of ring R.

(c) Prove that the set $S=\left\{g \in C[0,1] : g\left(\frac{1}{2}\right)=0\right\}$ is a subring of C[0,1].

[This question paper contains 4 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 515 **B**

Unique Paper Code : 62351201

Name of the Paper : Algebra

Name of the Course : **B.A. (Prog.)**

Semester : II

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt any **two** parts from each question.
3. **All** questions carry equal marks.

1. (a) Define subspace of a vector space. Show that the set $W = \{(a_1, a_2, a_3): a_1 - 2a_2 + a_3 = 0;\ a_1, a_2, a_3 \in R\}$ is a subspace of the vector space $R^3(R)$.

(b) Express the vector $v = (4,5)$ as a linear combination of the vectors $v_1 = (2,1)$, $v_2 = (1,2)$. Is the set $S = \{v, v_1, v_2\}$ linearly dependent or linearly independent?

(c) Define basis and dimension of a vector space. Do the vectors {(1, –1, 2), (–1, 2, –4), (–1, –1, 2)} in R^3 form a basis of $V = R^3(R)$. What is dim(V)?

2. (a) Find the rank of the following matrix

$$\begin{bmatrix} 1 & 1 & 0 & -2 \\ 2 & 0 & 2 & 2 \\ 4 & 1 & 3 & 1 \end{bmatrix}.$$

(b) Solve the following system of equations :

$$x + y + z = 2$$
$$x + 2y + 3z = 5$$
$$x + 3y + 6z = 11$$

(c) Show that the following matrix satisfies its characteristic equation :

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

3. (a) If $\cos\theta + 2\cos\varphi + 3\cos\psi = \sin\theta + 2\sin\varphi + 3\sin\psi = 0$, Prove that

$\cos 3\theta + 8\cos 3\varphi + 27\cos 3\psi = 18\cos(\theta + \varphi + \psi)$,

and $\sin 3\theta + 8\sin 3\varphi + 27\sin 3\psi = 18\sin(\theta + \varphi + \psi)$.

(b) Prove that

$$64\cos^7\theta = \cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos\theta.$$

(c) Solve the equation

$$z^5 + z^4 + z^3 + z^2 + z + 1 = 0.$$

4. (a) Find the sum of the cubes of the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$.

(b) Solve the equation

$$3x^4 - 25x^3 + 50x^2 - 50x + 12 = 0,$$

such that the product of two of the roots being 2.

(c) Solve the equation $x^3 - 9x^2 + 23x - 15 = 0$, being given that the roots are in A.P.

5. (a) If G is the set of all non-zero rational numbers with binary operation $*$ defined by $a * b = \frac{ab}{3}$, $a, b \in G$. Then prove that $(G, *)$ is an Abelian group.