Name of the Course	**: CBCS B.Sc. (Hons.) Mathematics**
Unique Paper Code	**: 32351101_OC**
Name of the Paper	**: C1-Calculus**
Semester	**: I**
Duration	**: 3 Hours**
Maximum Marks	**: 75**

Attempt any four questions out of the following. All questions carry equal marks.

1. Find the n^{th} derivative of $y = \frac{2x}{x^2+a^2}$. Also prove that $y_n = \frac{(-1)^n n!}{r^{n+1}} 2\cos(n+1)\theta$.

If $y = \cos(m \sin^{-1} x)$ then show that

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0.$$

2. Sketch the graph of $f(x) = x^4 - 4x^3 + 10$ by finding intervals of increase and decrease, critical points, relative extrema, inflection points and concavity for the given function.

Find the horizontal and vertical asymptotes to the graph of the function $f(x) = \frac{2x}{x^2-1}$.

Sketch the graph of the curve in polar coordinates of the curve $r = 1 + 2\cos\theta$.

3. Evaluate the following integrals

(i) $\int_0^{2\pi} \sin m\theta \cos n\theta \ d\theta$, (ii) $\int_0^{\frac{\pi}{3}} \sin^2 6\theta \cos^4 3\theta \ d\theta$.

Give reduction formula for $\int \text{cosec}^n \theta \, d\theta$ and find the value of $\int \text{cosec}^6 \theta \, d\theta$.

4. Use the method of cylindrical shells to find the volume of the solid generated when the region bounded by the hyperbola $y = \frac{1}{x}$ and the line $y = \frac{5}{2} - x$ is revolved about the y-axis.

Find the length of the curve $y = (x/2)^{\frac{2}{3}}$ from $x = 0$ to $x = 2$.

The arc of the parabola $y = x^2$ from (1,1) to (2,4) is rotated about y- axis. Find the area of the resulting surface.

5. Sketch the curve $(x-3)^2 = 6(y-2)$.

Sketch the curve $\frac{x^2}{15} - \frac{y^2}{3} = 1$, and label the vertices, foci and asymptotes.

Identify and sketch the curve $x^2 - xy + y^2 - 2 = 0$.

6. Find the limit: $\lim_{t\to 0} \frac{\sin 2t\hat{\imath} + t\hat{\jmath}}{t^2+t-1}$.

A shell is fired from the ground level with muzzle speed of 650 ft/s at an angle of 45°. An enemy gun 21,000 ft. away fires a shot 2 seconds later and the shells collided 45 ft. above the ground at the same speed. What are the muzzle speed (V_0) and angle of elevation (α) of the 2nd gun?

The acceleration vector of a moving particle is $A(t) = 18t^3\hat{\imath} + 3\hat{\jmath}$. Find the particles position vector as a function of t if $R(0) = \hat{\imath} + \hat{\jmath}$ and $V(0) = 1$.