(*b*) Show that $\Delta x^{(r)} = rhx^{(r-1)}$ where r being a positive integer and h being the interval of differencing.

(*c*) Show that $\Delta^n f(x) = ah^n(n!)$

where $f(x)$ is a rational integral function of the nth degree in x.

(*d*) Show that $\mu^2 y_x = \left(1+\frac{1}{4}\delta^2\right)y_x$.

(*e*) Show that the coefficients of Newton-Cotes formula are symmetric from both the ends. 1×5

7. (*a*) Derive Newton-Gregory formula for forward interpolation with equal interval.

(*b*) Use the method of separation of symbols to prove the following identity :

$$u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + ... + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n}$$

where $y = u_x$ is any function of x and $0 < x < 1$. 6,6

8. (*a*) Obtain Lagrange's interpolation formula in the form :

$$f(x) = \sum_{r=1}^{n} \frac{L(x)f(x_r)}{(x-x_r)L'(x_r)} = \sum_{r=1}^{n} L_r(x)f(x_r)$$

where, $L(x) = (x-x_1)(x-x_2).....(x-x_n)$

16/12/19 (PM)

This question paper contains **4+1** printed pages]

Roll No.

125

S. No. of Question Paper : **7505**

Unique Paper Code : **32371303** **J**

Name of the Paper : **Mathematical Analysis**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Q. No. **1** of Section-I is compulsory. Attempt *three* more questions from Section-I

Q. No. **6** of Section-II is compulsory. Attempt *two* more questions from Section-II

Use separate answer–books for Section-I and Section-II

Section-I

1. (*a*) Write the Supremum and Infimum of the following set :

$$S = \left\{1-\frac{1}{2^n}, n \in N\right\}$$

(*b*) (*i*) Give an example of an interval which is not a closed set.

(*ii*) Give an example of an open set which is not an interval.

(*c*) Write 'True' *or* 'False':

(*i*) A sequence can converge to more than one limit.

(*ii*) Every Cauchy sequence is bounded.

(*d*) Assuming that $n^{1/n} \to 1 \text{ as } n \to \infty$, show by applying Cauchy's *n*th root test that the series $\Sigma_{n=1}^{\infty} \left(n^{1/n} - 1\right)^n$ converges.

(*e*) Show that there is no real number k for which the equation $x^2 - 3x + k = 0$ has two distinct roots in [0, 1]. 2×5

2. (*a*) Show that the set of rational numbers is not ordered complete.

(*b*) Define neighbourhood of a point. If M and N are neighbourhoods of a point p, then prove that $M \cap N$ is also a neighbourhood of p. 6, 6

3. (*a*) Prove that a convergent sequence :

(*i*) is bounded and

(*ii*) has a unique limit.

(*b*) Prove that the sequence $<a_n>$ defined by the relation :

$$a_1 = 1, \; a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{(n-1)!} (n \geq 2)$$

converges. 6,6

4. (*a*) Prove that a necessary condition for a series Σu_n to converge is that :

$$\lim_{n \to \infty} u_n = 0$$

Is the condition sufficient ? Justify your answer.

(*b*) Examine the convergence of the following series :

(*i*) $\Sigma_{n=1}^{\infty} \dfrac{5^n}{n^2 + 5}$

(*ii*) $\dfrac{1}{2} + \dfrac{1.3}{2.4} + \dfrac{1.3.5}{2.4.6} + \dfrac{1.3.5.7}{2.4.6.8} + \ldots\ldots\ldots\ldots$ 6,6

5. (*a*) Let f be the function defined on [0, 1] by setting :

$$f(x) = \frac{1}{2^n}, \text{ when } \frac{1}{2^{n+1}} \leq x \leq \frac{1}{2^n}, n = 0,1,2,3,\ldots\ldots$$

$$f(x) = 0$$

Show that f is continuous except at the points $\frac{1}{2}, \frac{1}{2^2}, \ldots\ldots \frac{1}{2^n}, \ldots\ldots\ldots$ Describe the nature of discontinuity at each of these points.

(*b*) Obtain Maclaurin's series expansion of cos x. 6,6

Secton II

6. (*a*) Evaluate :

$\Delta^2(1 - x)(1 - 2x)(1 - 3x)$

where the interval of differencing is unity.

(*b*) Four equidistance values u_{-1}, u_0, u_1 and u_2 being given, a value is interpolated by Lagrange's interpolation formula. Show that it may be written in the form :

$$u_x = yu_o + xu_1 + \frac{y(y^2-1)}{3!}\Delta^2 u_{-1} + \frac{x(x^2-1)}{3!}\Delta^2 u_0.$$

where, $x + y = 1$. 6, 6

9. (*a*) Define Newton-Cotes formula and express :

$I = (x_n - x_0)\ \Sigma_0^n f(x_n) C_i^n$

(*b*) Solve any *two* of the following difference equations :

(*i*) $u_{x+1} - au_x = \sin bx$

(*ii*) $u_{x+1} - 2u_x = 2x$

(*iii*) $u_{x+2} + 2u_{x+1} + 4u_x = 0$ 6,6