

- (b) Show that  $\Delta x^{(r)} = r!hx^{(r-1)}$  where  $r$  being a positive integer and  $h$  being the interval of differencing.
- (c) Show that  $\Delta^n f(x) = ah^n(n!)$  where  $f(x)$  is a rational integral function of the  $n$ th degree in  $x$ .
- (d) Show that  $\mu^2 y_x = \left(1 + \frac{1}{4}\delta^2\right)y_x$ .
- (e) Show that the coefficients of Newton-Cotes formula are symmetric from both the ends. 1×5
7. (a) Derive Newton-Gregory formula for forward interpolation with equal interval.
- (b) Use the method of separation of symbols to prove the following identity :
- $$u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + \dots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n}$$
- where  $y = u_x$  is any function of  $x$  and  $0 < x < 1$ . 6,6
8. (a) Obtain Lagrange's interpolation formula in the form :

$$f(x) = \sum_{r=1}^n \frac{L(x)f(x_r)}{(x-x_r)L'(x_r)} = \sum_{r=1}^n L_r(x)f(x_r)$$

$$\text{where, } L(x) = (x-x_1)(x-x_2)\dots(x-x_n)$$

16/12/19 (P)

This question paper contains 4+1 printed pages]

Roll No.

--	--	--	--	--	--	--	--	--	--	--	--

S. No. of Question Paper : 7505

Unique Paper Code : 32371303

J

Name of the Paper : Mathematical AnalysisName of the Course : B.Sc. (H) Statistics

Semester : III

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Q. No. 1 of Section-I is compulsory. Attempt *three* more questions from Section-I

Q. No. 6 of Section-II is compulsory. Attempt *two* more questions from Section-II

Use separate answer-books for Section-I and Section-II

**Section-I**

1. (a) Write the Supremum and Infimum of the following set :

$$S = \left\{1 - \frac{1}{2^n}, n \in \mathbb{N}\right\}$$

- (b) (i) Give an example of an interval which is not a closed set.
- (ii) Give an example of an open set which is not an interval.

(c) Write 'True' or 'False':

- (i) A sequence can converge to more than one limit.
- (ii) Every Cauchy sequence is bounded.

(d) Assuming that  $n^{1/n} \rightarrow 1$  as  $n \rightarrow \infty$ , show by applying Cauchy's  $n$ th root test that the series  $\sum_{n=1}^{\infty} (n^{1/n} - 1)^n$  converges.

(e) Show that there is no real number  $k$  for which the equation  $x^2 - 3x + k = 0$  has two distinct roots in  $[0, 1]$ . 2x5

2. (a) Show that the set of rational numbers is not ordered complete.

(b) Define neighbourhood of a point. If  $M$  and  $N$  are neighbourhoods of a point  $p$ , then prove that  $M \cap N$  is also a neighbourhood of  $p$ . 6, 6

3. (a) Prove that a convergent sequence :

- (i) is bounded and
- (ii) has a unique limit.

(b) Prove that the sequence  $\langle a_n \rangle$  defined by the relation :

$$a_1 = 1, a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{(n-1)!} \quad (n \geq 2)$$

converges. 6,6

4. (a) Prove that a necessary condition for a series  $\sum u_n$  to converge is that :

$$\lim_{n \rightarrow \infty} u_n = 0$$

Is the condition sufficient ? Justify your answer.

(b) Examine the convergence of the following series :

(i)  $\sum_{n=1}^{\infty} \frac{5^n}{n^2 + 5}$

(ii)  $\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \frac{1.3.5.7}{2.4.6.8} + \dots$  6,6

5. (a) Let  $f$  be the function defined on  $[0, 1]$  by setting :

$$f(x) = \frac{1}{2^n}, \text{ when } \frac{1}{2^{n+1}} \leq x \leq \frac{1}{2^n}, n = 0, 1, 2, 3, \dots$$

$$f(x) = 0$$

Show that  $f$  is continuous except at the points

$\frac{1}{2}, \frac{1}{2^2}, \dots, \frac{1}{2^n}, \dots$  Describe the nature of discontinuity at each of these points.

(b) Obtain Maclaurin's series expansion of  $\cos x$ . 6,6

**Section II**

6. (a) Evaluate :

$$\Delta^2(1 - x)(1 - 2x)(1 - 3x)$$

where the interval of differencing is unity.