7. (a) Describe briefly the role and responsibilities of CSO. Name its main divisions.

(b) Discuss the main functions of the office of Registrar General of India.

(c) What is the background and objectives of COCSSO ?

5½, 4, 3

8. (a) Write a short note on historical perspective of statistical system in India.

(b) Show that a systematic sample has the same precision as the corresponding stratified random sample with one unit per stratum if $\rho_{wst} = 0$, the notation has its usual meaning.

5½, 7

This question paper contains **4** printed pages]

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(76)

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all,

selecting *three* from each Section.

Section I

1. (a) What are the main steps involved in conducting a sample survey ? Discuss briefly.

(b) If (X_i, Y_i), $i = 1, 2, ..., N$ are the pairs of variates defined for every unit of the population of size N and $\bar{x}_n$ and $\bar{y}_n$ are the corresponding means of a srswor of size n, then prove that : (5, 7½)

$$\text{cov}(\bar{x}_n, \bar{y}_n) = \text{cov}(\bar{x}_n, y_1) = \frac{N-n}{n(N-1)} \text{cov}(X, Y).$$

P.T.O.

2. (a) Derive the variance of the ratio estimator of the population mean to the first approximation. Obtain the condition under which it is more efficient than SRSWOR.

   (b) In a simple random sample without replacement, prove that sample mean square is an unbiased estimator of population mean square.   7½, 5

3. (a) Give an example where two-stage sampling scheme may be adopted. For equal size first-stage units, obtain an estimator for population mean in two-stage sampling and its variance.

   (b) Prove that the mean of cluster means $\bar{\bar{y}}$ is an unbiased estimator of population mean and derive its variance in terms of intra-class correlation between the elements of a cluster.   5½, 7

4. (a) Show that the regression estimator is always more efficient than ratio estimator. Derive the condition under which the ratio and regression estimators are equally precise.

   (b) Derive the expression for the variance of the sample mean. Find the value of n in order that this variance of the mean in SRSWOR may be exactly half the variance of the mean of a random sample of the same size taken with replacement.   5½, 7

Section II

5. (a) Explain the concept of post-stratification. Show that, for large samples, the post-stratification is as precise as stratified sampling with proportional allocation.

   (b) If the population consists of a linear trend, then in usual notations show that,

   $$V(\bar{y}_{st}) \le V(\bar{y}_{sys}) \le V(\bar{y}_n)_{SRS}$$

   Also, show that the weighted sample mean in systematic sampling with Yates' end corrections provide the exact population mean.   5½, 7

6. (a) Obtain the estimated gain in precision due to arbitrary stratification over simple random sampling without replacement for estimating the populations mean $\bar{Y}_N$.

   (b) With two strata, a surveyor would like to have $n_1 = n_2$ for administrative convenience instead of using the values given by Neyman's allocation. If $V(\bar{y}_{st})$ and $V(\bar{y}_{st})_{opt}$ denote the variances given by $n_1 = n_2$ and Neyman's allocation respectively, show that the fractional increase in the variance is

   $$\frac{V(\bar{y}_{st}) - V(\bar{y}_{st})_{opt}}{V(\bar{y}_{st})_{opt}} = \left(\frac{r-1}{r+1}\right)^2$$

   Where $r = n_{1(opt)}/n_{2(opt)}$ and f.p.c are ignored. 6½, 6

P.T.O.