2-12-19 (Morn.)

This question paper contains 7 printed pages]

Roll No. | | | | | | | | | | | |

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S. No. of Question Paper : **7506**

Unique Paper Code : **32371501** **J**

Name of the Paper : **Stochastic Processes and Queueing Theory**

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Section **I** is compulsory.

Attempt *four* more questions, selecting *two* questions from each of Sections **II** and **III**.

Use of simple calculator is allowed.

Section I

1. Attempt any *five* parts :

   (*a*) If X has a zero truncated Poisson distribution with zero class missing then obtain the probability generating function of X.

P.T.O.

(*b*) Let X be a non-negative integral valued random variable with $P(X = k) = p_k$; $k = 0, 1, 2 \ldots$, and probability generating function $P(s)$. Find the generating function of $P(X > k + 1)$.

(*c*) Consider the process $X(t) = \sum_{r=1}^{k} (A_r \cos\theta_r t + B_r \sin\theta_r t)$ where A_r, B_r are uncorrelated random variables with mean 0 and variance σ^2 and θ_r are constants. Is the process $\{X(t), t \in T\}$ covariance stationary ?

(*d*) In a series of Bernoulli trials with probability of success p, let X denote the number of failures preceding the first success and Y, the number of failures following the first success and preceding the second success, then show that the p.g.f. of $Z = X + Y$ is $P(s,s) = \left(\frac{p}{1-qs}\right)^2$. Hence, obtain an expression for $P(Z = r)$.

7. (*a*) An airline organization has one reservation clerk on duty in its local branch. The clerk handles information regarding passenger reservations and flight timings. Assume that the number of customers arriving during any given period is in a Poisson fashion with an arrival rate of 8 per hour and that the reservation clerk can serve a customer in 6 minutes on an average with an exponentially distributed service time. Calculate the following :

(*i*) Probability that the system is busy,

(*ii*) Average time that a customer spends in the system,

(*iii*) Average queue length.

(*b*) For the model (M/M/1) : (N/FIFO), derive the steady state equations and find the probability distribution of number of customers in the system. Hence obtain the expected queue length. 7,8