4. (*a*) Let x_1 and x_2 be the number of units possessing an attribute in random samples of sizes n_1 and n_2 respectively drawn from two populations. Proportions (unknown) possessing the attribute in the two populations are P_1 and P_2. If $p_1 = \frac{x_1}{n_1}$ and $p_2 = \frac{x_2}{n_2}$, derive a test of significance of difference between P_1 and P_2.

(*b*) Discuss type I and type II errors with examples. Also explain critical region and level of significance with an illustration. 6,6

Section II

5. (*a*) Let $x_1, x_2, \ldots, x_n$ be independent observations from a normal population with mean μ and variance σ^2. Let $\bar{x}$ and s^2 be the sample mean and sum of squares of the deviations from the mean respectively. Let x be one more observation independent of previous ones. Show that :

$$\frac{x-\bar{x}}{s}\left[\frac{n(n-1)}{n+1}\right]^{1/2}$$

has a student's t distribution with $(n-1)$ d.f.

3/12/2019 (Morning)

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | |

S. No. of Question Paper : **7503**

29

ique Paper Code : **32371301** **J**

Name of the Paper : **Sampling Distributions**

Name of the Course : **B.Sc. (H) Statistics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory.

Attempt *six* questions in all by selecting at least *two* questions from each section.

1. Attempt any *five* parts :

(*a*) Examine if WLLN holds for the sequence $\{S_n\}$, where $S_n = X_1 + X_2 + X_3 \ldots\ldots + X_n$. $\{X_k\}$ are independent and identically distributed random variables with mean μ and finite variance.

(*b*) Show that the sample variance s^2 is a biased estimator of the population variance σ^2.

(*c*) Let $X \sim B(n, p)$. Using Chebychev's inequality, find an upper bound on $P(X > \alpha n)$ where $p < \alpha < 1$. Evaluate the bound for $p = \frac{1}{2}$, $\alpha = \frac{3}{4}$.

(*d*) State and prove the additive property of chi square distribution.

(*e*) Let X_1 and X_2 be a random sample of size 2 from N(0, 1). Find the distribution of :

(*i*) $(X_1 - X_2)/\sqrt{2}$

(*ii*) $(X_1 + X_2)^2/(X_2 - X_1)^2$.

(*f*) Define convergence with probability one and convergence in distribution.

(*g*) Consider a random sample of size n from a population with pdf $f(x)$, where $f(x)$ is symmetric at $x = \mu$. Show that $f_r(\mu + x) = f_{n-r+1}(\mu - x)$, where $f_r(x)$ is the p.d.f of the r^{th} order statistic. 5×3=15

Section I

2. (*a*) Find the pdf of r^{th} order statistic. Let $X_1, X_2, \ldots, X_n$ are *i.i.d* variates with distribution function given by :

$$F(x) = x^{\alpha},\ i = 1, 2, \ldots\ n,\ 0 < x < 1,\ \alpha > 0$$

Show that, $X_{(i)}/X_{(n)}$, $(i = 1, 2, 3 \ldots, n^{-1})$ and $X_{(n)}$ are independently distributed.

(*b*) State and prove Lindeberg-Levy central limit theorem for *i.i.d.* random variables. 6,6

3. (*a*) State Chebychev's Inequality and explain its significance.

Let X have a pdf :

$$f(x) = \frac{1}{2\sqrt{3}}, \qquad -\sqrt{3} < x < \sqrt{3}$$

Find the actual probability $P[|X - \mu| \geq (3/2)\sigma]$ and compare with the upper bound obtained by Chebychev's inequality.

(*b*) If X_i can have only two values i^{α} and $-i^{\alpha}$, with equal probabilities, show that the weak law of large numbers can be applied to the independent random variables $X_1, X_2, \ldots$, if $\alpha < \frac{1}{2}$. 6,6 (*b*) Show that the chi square test involving two sample proportions is equivalent to a large sample significance test of difference in proportions. 6,6

6. (*a*) If X_1 and X_2 are independently distributed, each as chi square variate with 2 degrees of freedom, find the p.d.f. of $Y = (X_1 - X_2)/2$.

(*b*) Define the F statistic. Prove that if $n_1 = n_2$, the median of F distribution is at $F = 1$ and the quartiles Q_1 and Q_3 satisfy the condition $Q_1Q_3 = 1$. 6,6

7. (*a*) Obtain mean deviation about mean for *t*–distribution with *n* d.f.

(*b*) Show that mean of the $F(n_1, n_2)$ distribution is independent of n_1.

(*c*) If X is a chi-square variate with *n* d.f., then prove that for large *n*, $\sqrt{2X} \sim N(\sqrt{2n}, 1)$. 4,4,4

8. (*a*) Let $X_1, X_2, \ldots. X_n$ be a random sample from a normal population with mean μ and variance σ^2. Then, show that $\bar{X}$ and $\frac{ns^2}{\sigma^2} = \sum_{i=1}^{n} \left(\frac{x_i = \bar{x}}{\sigma}\right)^2$ are independently distributed. Also find the distribution of $\frac{ns^2}{\sigma^2}$.

(*b*) Define Fisher's *t*–statistic. Compare the graph of the *t*–distribution with that of the standard normal distribution. 8,4