

4. (a) Let x_1 and x_2 be the number of units possessing an attribute in random samples of sizes n_1 and n_2 respectively drawn from two populations. Proportions (unknown) possessing the attribute in the two populations are P_1 and P_2 . If $p_1 = \frac{x_1}{n_1}$ and $p_2 = \frac{x_2}{n_2}$, derive a test of significance of difference between P_1 and P_2 .
- (b) Discuss type I and type II errors with examples. Also explain critical region and level of significance with an illustration. (66)

Section II

5. (a) Let x_1, x_2, \dots, x_n be independent observations from a normal population with mean μ and variance σ^2 . Let \bar{x} and s^2 be the sample mean and sum of squares of the deviations from the mean respectively. Let x be one more observation independent of previous ones.

Show that :

$$\frac{x - \bar{x}}{s} \left[\frac{n(n+1)}{n+1} \right]^{1/2}$$

has a student's t distribution with $(n - 1)$ d.f.

3/12/2019 (Morning)

This question paper contains 4+2 printed pages]

Roll No.

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S. No. of Question Paper : 7503

(29)

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Name of the Paper : Sampling Distributions

Name of the Course : B.Sc. (H) Statistics

Semester : III

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory.

Attempt six questions in all by selecting

at least two questions from each section.

1. Attempt any five parts :

(a) Examine if WLLN holds for the sequence $\{S_n\}$, where

$$S_n = X_1 + X_2 + X_3 + \dots + X_n, \{X_k\} \text{ are independent}$$

and identically distributed random variables with mean

μ and finite variance.

P.T.O.

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(b) Show that the sample variance s^2 is a biased estimator

of the population variance σ^2 .

(c) Let $X \sim B(n, p)$. Using Chebychev's inequality, find

an upper bound on $P(X > \alpha n)$ where $p < \alpha < 1$.

Evaluate the bound for $p = 1/2, \alpha = 3/4$.

(d) State and prove the additive property of chi square

distribution.

(e) Let X_1 and X_2 be a random sample of size 2 from

$N(0, 1)$. Find the distribution of :

(i) $(X_1 - X_2)/\sqrt{2}$

(ii) $(X_1 + X_2)^2/(X_2 - X_1)^2$.

(f) Define convergence with probability one and

convergence in distribution.

(g) Consider a random sample of size n from a population

with pdf $f(x)$, where $f(x)$ is symmetric at $x = \mu$. Show

that $f_r(\mu + x) = f_{n-r+1}(\mu - x)$, where $f_r(x)$ is the p.d.f

of the r^{th} order statistic. 5×3=15

Section I

2. (a) Find the pdf of r^{th} order statistic. Let X_1, X_2, \dots, X_n

are *i.i.d* variates with distribution function given by :

$$F(x) = x^\alpha, \quad i = 1, 2, \dots, n, \quad 0 < x < 1, \quad \alpha > 0$$

Show that, $X_{(i)}/X_{(n)}$, ($i = 1, 2, 3, \dots, n-1$) and $X_{(n)}$ are independently distributed.

(b) State and prove Lindeberg-Levy central limit theorem for *i.i.d* random variables. 6,6

3. (a) State Chebychev's Inequality and explain its significance.

Let X have a pdf :

$$f(x) = \frac{1}{2\sqrt{3}}, \quad -\sqrt{3} < x < \sqrt{3}$$

Find the actual probability $P[|X - \mu| \geq (3/2)\sigma]$ and compare with the upper bound obtained by Chebychev's inequality.

(b) If X_i can have only two values r^α and $-r^\alpha$, with equal probabilities, show that the weak law of large numbers can be applied to the independent random variables X_1, X_2, \dots , if $\alpha < 1/2$. 6,6