8. (*a*) A box contains 6 red, 4 white and 5 black balls. A person draws 4 balls from the box at random. Find the probability that among the balls drawn, there is at least one ball of each color.

(*b*) In a test, a candidate either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that is answer is correct, given that he copied it is 1/8. Find the probability that he knew the answer to the question, given that he correctly answered it. 6, 6

7/12/19

This question paper contains 8 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **8601**

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Unique Paper Code : **32371101** J

Name of the Paper : **Descriptive Statistics**

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **6** questions in all.

Question No. **1** is compulsory.

Attempt **5** more questions,

selecting at least **2** questions from each section.

Use of simple calculator is allowed.

1. (*a*) Fill in the blanks :

(*i*) The geometric mean of 3, 9, 27 and 81 is ....

(*ii*) The total number of class frequencies of all orders, for n attributes is ....

(*iii*) The standard deviation is the least value of ....

(*iv*) If γ_2 is greater than zero, the distribution is ....

(*v*) A letter is selected from the word 'UNIVERSITY'. The probability that it is a vowel is ....

(*b*) Mean, median and mode of 10 numbers are 50, 52 and 55 respectively. The value of the largest number is 100. It was later found that it is actually 110. Find the corrected mean, median and mode.

(*c*) Let P(E) = 0.7, P(F) = 0.5 and $P\left(\overline{E} \cap \overline{F}\right) = 0.1$. Find $P(E \cap F)$ and comment on the independence of the events E and F.

7. (*a*) Let X_1 and X_2 be two independent random variables having the same distribution with p.d.f.

$$f(x) = e^{-x}, \quad x \geq 0.$$

(*i*) Find the joint p.d.f. of (X_1, X_2),

(*ii*) Find joint p.d.f. of transformed variables :

$$Y_1 = X_1 + X_2 \text{ and } Y_2 = \frac{X_1}{X_1 + X_2},$$

(*iii*) Comment on the independence of Y_1 and Y_2.

(*b*) The odds against the wife who is 40 years old living till she is 70 is 8 : 5 and odds against her husband now 50 living till he is 80 is 4 : 3. Find the probability that 30 years hence,

(*i*) both will be alive,

(*ii*) only one will be alive, and

(*iii*) at least one will be alive. 6, 6 (*b*) The p.d.f. of a random variable X is given by :

$$f(x) = x^2/18; \quad -3 \le x \le 3.$$

Find the p.d.f. of transformed variable $Y = 4X^2 - 3$. 6, 6

Section B

6. (*a*) State and prove addition theorem of probability for three events A_1, A_2 and A_3 and hence, show that :

(*i*) $P(A_1 \cup A_2 \cup A_3) \le P(A_1) + P(A_2) + P(A_3)$,

(*ii*) $P(A_1 \cap A_2 \cap A_3) \ge P(A_1) + P(A_2) + P(A_3) - 2$.

(*b*) Define independent events, pairwise independence, and mutual independence. An urn contains four tickets marked with numbers 112, 121, 211 and 222 respectively. One ticket is drawn at random. Let A_i ($i = 1, 2, 3$) be the events that *i*th digit of the number of the ticket drawn is 1. Discuss the independence of the events A_1, A_2 and A_3. 6, 6

(*d*) Examine the consistency of the following data :

N = 200; (A) = 120, (B) = 100, (AB) = 10,

the symbols having their usual meaning.

(*e*) Let X be a random variable with c.d.f.

$$F(x) = \begin{cases} k\left(1 - e^{-x}\right)^2; & x > 0 \\ 0 \quad ; & \text{otherwise.} \end{cases}$$

Find the value of constant k and $P(X < 3)$.

(*f*) A coin is tossed until a head appears. Find the expected number of tosses required. 5×1, 2, 2, 2, 2, 2

Section A

2. (*a*) Show that in a discrete series if deviations $x_i = X_i - M$, are small compared with the value of the mean M so that $(x/M)^3$ and higher powers of (x/M) are neglected, then

$$\text{Mean}\left(\sqrt{X}\right) = \sqrt{M}\left(1 - \frac{\sigma^2}{8M^2}\right) \text{ approximately}$$

where, σ^2 is the variance.

(*b*) Let r be the range and s be the standard deviation of a set of observations $x_1, x_2, ..., x_n$. Prove that $s \leq r$. Further prove that :

$$S \leq r\left(\frac{n}{n-1}\right)^{\frac{1}{2}}$$

where,

$$S = \left[\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}\right)^2\right]^{\frac{1}{2}}.$$

6, 6

3. (*a*) Let X be a random variable with p.d.f. :

$$f(x) = k\,(1 - |x - b|/a);\ b - a < x < b + a,$$

where a, b and k are constants. Find k. Obtain mean and variance of X.

(*b*) Show that for n attributes $A_1, A_2, ..., A_n$:

$$(A_1A_2A_3\ ....\ A_n) \geq (A_1) + (A_2) + (A_3) + ...\ + (A_n) - (n-1)N$$

where, N is the total number of observations. 6, 6

4. (*a*) In a frequency table, the upper boundary of each class interval has a constant ratio to the lower boundary. Show that the geometric mean G may be expressed by the following formula :

$$\log G = x_0 + \frac{c}{N}\sum_i f_i(i-1)$$

where, x_0 is the logarithm of the mid value of the first interval and c is the logarithm of the ratio between upper and lower boundaries.

(*b*) Define Yule's coefficient of association (Q) and coefficient of colligation (Y). Prove that $Q = 2Y/(1 + Y^2)$. Also, find the range of Q. Under what conditions are the extreme values attained ? 6, 6

5. (*a*) A deck of n numbered cards are thoroughly shuffled and the cards are inserted into n numbered cells one by one. If the card 'i' falls in the cell i, it is counted as a match, otherwise not. Find the mean of total number of such matches.