

- (c) Find the cumulative distribution function for the following pdf :

$$f(x) = \begin{cases} 1/3 & 0 < x < 1 \\ 1/3 & 2 < x < 4 \\ 0 & \text{elsewhere} \end{cases}$$

Also find the median.

3. (a) Let X have the mgf

$$M(t) = e^{t^2/2}, -\infty < t < \infty$$

Find  $E(X^{2k})$  and  $E(X^{2k-1})$ , for  $k = 1, 2, 3, \dots$

- (b) Show by stating all the conditions that the Binomial distribution can be approximated to the Poisson distribution.

- (c) Let X have the exponential pdf,  $f(x) = \theta^{-1} \exp \{-x/\theta\}$ ,  $0 < x < \infty$ , zero elsewhere. Find the moment generating function of X, and hence, the mean, and the variance of X.

4. (i) Let  $X_1$  and  $X_2$  have the joint pdf

$$f(x_1, x_2) = 15x_1^2x_2 \text{ if } 0 < x_1 < x_2 < 1 \\ = 0 \text{ elsewhere}$$

Find the marginal pdf of  $X_1$  and  $X_2$  and compute  $P(X_1 + X_2 \leq 1)$ .

This question paper contains 4+2 printed pages]

Roll No.

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S. No. of Question Paper : 2710

Unique Paper Code : 32357607

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Name of the Paper

: Probability Theory & Statistics

Name of the Course

: B.Sc. (Hons.) Mathematics : DSE-3

Semester

: VI

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

In all there are six questions.

Question No. 1 is compulsory and it contains seven parts of 3 marks each, out of which any five parts are to be attempted.

In Question Nos. 2 to 6, attempt any two parts from three parts.

Each part carries 6 marks.

Use of scientific calculator is allowed.

1. (i) If  $C_1, C_2$  and  $C_3$  are events in C, then prove that

$$p_1 \geq p_2 \geq p_3$$

where  $p_1 = P(C_1) + P(C_2) + P(C_3)$ ,

$p_2 = P(C_1 \cap C_2) + P(C_2 \cap C_3) + P(C_1 \cap C_3)$ , and

$p_3 = P(C_1 \cap C_2 \cap C_3)$ .

P.T.O.

(ii) Given the cumulative distribution function

$$F(x) = 0 \text{ if } x < -1$$

$$= (x + 2)/4, \text{ if } -1 \leq x < 1, \text{ and}$$

$$= 1 \text{ if } 1 \leq x,$$

Compute :

(i)  $P(-1/2 < X \leq 1/2)$ ;

(ii)  $P(X = 1)$ .

(iii) Let pmf  $p(x)$  be positive at  $x = -1, 0, 1$  and zero elsewhere.

If  $p(0) = \frac{1}{4}$ , find  $E(X^2)$ .

(iv) If the random variable  $X$  has a binomial distribution with the parameters  $n$  and  $\theta$ , then compute the variance,  $\sigma^2$ , of  $X$ .

(v) Let  $F(x, y)$  be the distribution function of  $X$  and  $Y$ . For all real constants  $a < b, c < d$ , show that

$$P(a < X \leq b, c < Y \leq d) = F(b, d) - F(b, c)$$

$$- F(a, d) + F(a, c)$$

(vi) Let  $f_{1/2}^{(x_1/x_2)} = \begin{cases} \frac{c_1 x_1}{x_2^2}, & 0 < x_1 < x_2, 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$

be the conditional pdf of  $X_1$  given  $X_2 = x_2$ .

Also let  $f_2(x_2) = \begin{cases} c_2 x_2^4, & 0 < x_2 < 1 \\ 0, & \text{elsewhere} \end{cases}$

be the marginal pdf of  $X_2$ .

Determine  $C_1$  &  $C_2$  and hence the joint pdf of  $X_1$  and  $X_2$ .

(vii) Prove that  $P_{i,j}^{n+m} = \sum_{k=0}^{\infty} P_{i,k}^n P_{k,j}^m$ , for all  $n, m$  and all  $i, j$ .

2. (a) Let  $\{C_n\}$  be a decreasing sequence of events, then show that

$$\lim_{n \rightarrow \infty} P(C_n) = P(\lim_{n \rightarrow \infty} C_n) = P(\bigcap_{n=1}^{\infty} C_n)$$

(b) In a lot of 50 light bulbs, there are 2 bad bulbs. An inspector examines five bulbs, which are selected at random and without replacement.

(i) Find the probability of at least one defective bulb among the five.

(ii) How many bulbs should be examined so that the probability of finding at least one bad bulb exceeds  $1/2$  ?