

- (c) Set up the Gauss-Seidel iteration scheme to solve the system of equations :

$$6x_1 - 2x_2 + x_3 = 11$$

$$-2x_1 + 7x_2 + 2x_3 = 5$$

$$x_1 + 2x_2 - 5x_3 = -1$$

Take the initial approximation as $X^{(0)} = (1, 0, 0)$ and do three iterations. 6.5

4. (a) Construct the Lagrange form of the interpolating polynomial from the following data : 6

x	-1	0	1
$f(x) = e^x$	e^{-1}	e^0	e^1

- (b) Construct the divided difference table for the following data set and then write out the Newton form of the interpolating polynomial :

x	-1	0	1	2
y	5	1	1	11

Hence, estimate the value of $f(0.5)$. 6

4/12/19 (M)

This question paper contains 4+2 printed pages]

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S. No. of Question Paper : 7941 39

Unique Paper Code : 32357501 J

Name of the Paper : Numerical Methods

Name of the Course : B.Sc. (H) Mathematics : DSE-1

Semester : V

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the six questions are compulsory.

Attempt any two parts from each question.

Marks are indicated against each question.

Use of Non-Programmable Scientific Calculator is allowed.

1. (a) Find a root of the equation $x \sin(x) + \cos(x) = 0$ using Newton-Raphson Method for starting approximation $x_0 = \pi$. 6.5

P.T.O.

(b) A real root of the equation $x^3 - 5x + 1 = 0$ lies in $]0, 1[$. Perform three iterations of Bisection Method to obtain the root. 6.5

(c) Prove that fixed point method converges at a linear rate using $g(x) \equiv x^2 - 2x - 3 = 0$ and starting approximation $x_0 = 4$. 6.5

2. (a) Using Secant Method, find a real root of the equation $xe^x - 1 = 0$ correct upto four decimal places. Given the root lies between 0 and 1, perform three iterations. 6

(b) Consider the function $g(x) = 1 + x - \frac{x^3}{8}$. Analytically verify that this function has unique fixed point on the real line. Perform three iterations starting at $x_0 = 0$ to locate the fixed point. 6

(c) Evaluate order of convergence of Newton Method numerically using $f(x) \equiv x^5 + 2x - 1 = 0$ and initial approximation $x_0 = \frac{1}{2}$. 6

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 2 & 7 & 5 \\ 6 & 20 & 10 \\ 4 & 3 & 0 \end{bmatrix}$$

and use it to solve the system $AX = [0 \ 4 \ 1]^T$. 6.5

(b) Set up the Gauss-Jacobi iteration scheme to solve the system of equations :

$$4x_1 + x_2 + 2x_3 = 4$$

$$3x_1 + 5x_2 + x_3 = 7$$

$$x_1 + x_2 + 3x_3 = 3$$

Take the initial approximation as $X^{(0)} = (0, 0, 0)$ and do three iterations. 6.5

- (c) Find the maximum value of the step size h that can be used in the tabulation of $f(x) = e^x$ on the interval $[0, 1]$ so that the error in the linear interpolation of $f(x)$ is less than 5×10^{-4} . 6

5. (a) Define the forward difference operator Δ , the central difference operator δ and the averaging operator μ .

Prove that :

$$(i) \quad \Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}$$

$$(ii) \quad \mu = \left(1 + \frac{1}{2}\Delta\right)(1 + \Delta)^{-\frac{1}{2}}. \quad 6.5$$

- (b) Use the formula

$$f'(x_0) \approx \frac{f(x_0) - f(x_0 - h)}{h}$$

to approximate the derivative of $f(x) = \sin x$ at $x_0 = \pi$ taking $h = 1, 0.1, 0.01$. What is the order of approximation? 6.5

- (c) Derive the following forward difference approximation for the second derivative : 6.5

$$f''(x_0) \approx \frac{f(x_0) - 2f(x_0 + h) + f(x_0 + 2h)}{h^2}$$

6. (a) Define degree of precision of a quadrature rule. Show that the degree of precision of the Trapezoidal rule is 1. 6

- (b) Approximate the value of the integral $\int_0^1 \frac{1}{1+x^2} dx$ using

Simpson's 1/3rd rule. 6

- (c) Use Euler's method to approximate the solution of the initial value problem.

$$x' = tx^3 - x, x(0) = 1, 0 \leq t \leq 1 \text{ taking 4 steps. } 6$$