-02.12/19 (Mon.)

2|12|2019 (M)

[This question paper contains 7 printed pages]

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Your Roll No. : ....

Sl. No. of Q. Paper : **7466** **J**

Unique Paper Code : 32351501

Name of the Course : **B.Sc.(Hons.) Mathematics**

Name of the Paper : Metric Spaces

Semester : V

Time : 3 Hours ***Maximum Marks : 75***

Instructions for Candidates :

(a) Write your Roll No. on the top immediately on receipt of this question paper.

(b) Attempt any **two** parts from each question.

1. (a) Define a metric space. Let $p \geq 1$. Define $d_p : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ as $d_p(x,y) = \left(\sum_{i=1}^{n} |x_i - y_i|^p\right)^{1/p}$, $x\ (x_1, x_2, \ldots, x_n)$, $y = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n$. Show that $(\mathbb{R}^n, d_p)$ is a metric space.

6.5

P.T.O.

(b) When is a metric space said to be complete ? Is discrete metric space complete ? Justify. 6.5

(c) Let (X, d) be a metric space. Define $d_1 : X \times X \to \mathbb{R}$ by $d_1(x,y) = \frac{d(x,y)}{1+d(x,y)}$, for all $x, y \in X$. Prove that d_1 is a metric on X and d_1 is equivalent to d. 6.5

2. (a) Prove that every open ball in a metric space (X, d) is an open set in (X, d). What about the converse ? Justify. 6

(b) Define a homeomorphism from a metric space (X, d_1) to a metric space (Y, d_2). Show that the function $f : \mathbb{R} \to \,]-1, 1[$ defined by $f(x) = \frac{x}{1+|x|}$ is a homeomorphism. 6

(b) Let (X, d) be a metric space and Y be a subset of X. If Y is compact subset of (X, d), then prove that Y is closed. 6.5

(c) Let f be a continuous function from a compact metric space (X, d_1) to a metric space (Y, d_2). Prove that f is uniformly continuous on X. 6.5