

7467

5. (a) State the class equation for finite groups. Find conjugacy classes of the quaternion group Q_8 and hence verify the class equation for Q_8 . $2+3+1.5$
- (b) Let p be a prime and P be a group of prime power order p^α for some $\alpha \geq 1$. Then prove that P has a non trivial centre. Deduce that a group of order p^2 is an Abelian group. $4+2.5$
- (c) Let G be a non-Abelian group of order 231. Then prove that a Sylow 11-subgroup is normal and is contained in the centre of G . $2.5+4$
6. (a) Let G be a group of order pq such that $p < q$ and p does not divide $(q - 1)$. Then prove that G is a cyclic group. Hence deduce that a group of order 33 is cyclic. $4.5+2$
- (b) Define a simple group. Prove that groups of order 72 and 56 are not simple. $1 + 2.5 + 3$
- (c) Let G be a group such that $|G|=2n$, where $n \geq 3$ is an odd integer. Then prove that G is not simple. 6.5

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[This question paper contains 4 printed pages]

Your Roll No. :

Sl. No. of Q. Paper : 7467 J

Unique Paper Code : 32351502

Name of the Course : B.Sc.(Hons.)
Mathematics

Name of the Paper : Group Theory - II

Semester : V

Time : 3 Hours Maximum Marks : 75

Instructions for Candidates :

- (a) Write your Roll No. on the top immediately on receipt of this question paper.
- (b) Attempt any **two** parts from each question.
- (c) All questions carry equal marks.
1. (a) Let $\text{Inn}(D_8)$ denotes the group of inner automorphisms on the dihedral group D_8 of order 8. Find $\text{Inn}(D_8)$. 6
- (b) Define inner automorphism of a group G induced by $g \in G$. Then prove that the set $\text{Inn}(G)$ of all inner automorphism of a group G is a normal subgroup of the group $\text{Aut}(G)$ of all automorphisms of G . $2+4$