

7464

(ii) Show that Z , the group of integers under addition is not isomorphic to Q , the group of rationals under addition. $6 \times 2 = 12$

6. (a) Let ϕ be a group homomorphism from a group G to a group G^* then prove that :

(i) $|\phi(x)|$ divides $|x|$, for all x in G .

(ii) ϕ is one-one if and only if $|\phi(x)| = |x|$, for all x in G .

(b) State and prove the Third Isomorphism Theorem.

(c) (i) Let G be a group. Prove that the mapping $\phi(g) = g^1$, for all $g \in G$, is an isomorphism on G if and only if G is Abelian.

(ii) Determine all homomorphisms from Z_n to itself. $6.5 \times 2 = 13$

9/12/19 (m)

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[This question paper contains 4 printed pages]

Your Roll No. :

Sl. No. of Q. Paper : 7464 J

Unique Paper Code : 32351302

Name of the Course : B.Sc.(Hons.)
Mathematics

Name of the Paper : Group Theory - I

Semester : III

Time : 3 Hours Maximum Marks : 75

Instructions for Candidates :

- Write your Roll No. on the top immediately on receipt of this question paper.
- Attempt any **two** parts from each question.
- All questions carry equal marks.

1. (a) Let $G = \left\{ \begin{bmatrix} a & 0 \\ 0 & 0 \end{bmatrix} : a \in \mathbb{R}, a \neq 0 \right\}$. Show that G is a group under matrix multiplication.

(b) Let G be a group and H be a subset of G . Prove that H is a subgroup of G if $a, b \in H \Rightarrow ab^{-1} \in H$. Hence prove that $H = \{A \in G : \det A \text{ is a power of } 3\}$ is a subgroup of $GL(2, \mathbb{R})$.