

14. Identify and sketch the curve :

$$x^2 + 4xy - 2y^2 - 6 = 0.$$

Section IV

Attempt any *four* questions from Section IV.

15. Evaluate :

$$\lim_{t \rightarrow 0^+} \left[\frac{\sin 3t}{\sin 2t} \hat{i} + \frac{\log(\sin t)}{\log(\tan t)} \hat{j} + (t \log t) \hat{k} \right].$$

16. The acceleration of a moving particle is $\vec{A}(t) = 24t^2 \hat{i} + 4 \hat{j}$. Find the particle's position as a function of t if $\vec{R}(0) = \hat{i} + 2 \hat{j}$ and $\vec{v}(0) = 0$.

17. If a shot putter throws a shot from a height of 5 ft with an angle of 46° and initial speed of 25 ft/sec, what is the horizontal distance of the throw ?

18. Find $\vec{T}(t)$, $\vec{N}(t)$ and $\vec{B}(t)$ for $\vec{r}(t) = \cos t \hat{i} + \sin t \hat{j} + \hat{k}$ at $t = \frac{\pi}{4}$.

19. Show that the curvature of the polar curve $r = e^{\alpha\theta}$ is inversely proportional to r .

This question paper contains 4 printed pages]

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S. No. of Question Paper : 8597

(60)

Unique Paper Code : 32351101

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Name of the Paper : Calculus

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : I

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All sections are compulsory.

All questions carry equal marks.

Use of non-programmable scientific calculator is allowed.

Section I

Attempt any *four* questions from Section I.

1. State Leibnitz's theorem for finding n th derivative of product of two functions. If $y = a \cos(\ln x) + b \sin(\ln x)$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

2. Evaluate the following limit :

$$\lim_{x \rightarrow 0^+} x^{\sin x}$$

3. Find the intervals of increase and decrease of the following function, discuss its concavity and then sketch its graph

$$y = (x+1)^2(x-5).$$

4. Sketch the graph of the polar curve $r = 3 \cos 2\theta$.
5. A manufacturer estimates that when 'x' units of a particular commodity are produced each month, the total cost (in dollars) will be $C(x) = \frac{1}{8}x^2 + 4x + 200$ and units can be sold at a price of $p(x) = 49 - x$ dollars per unit. Determine the price that corresponds to the maximum profit.

Section II

Attempt any *four* questions from Section II.

6. Find a reduction formula for $\int \operatorname{cosec}^n x dx$, $n \geq 2$ is an integer. Evaluate $\int \operatorname{cosec}^4 x dx$.
7. Find the volume of the solid generated when the region bounded by $y = \sqrt{25 - x^2}$, $y = 3$, is revolved about the x-axis.

8. The base of a certain solid is enclosed by $y = \sqrt{x}$, $y = 0$, and $x = 4$. Every cross-section perpendicular to the x-axis is a semicircle with its diameter across the base. Find the volume of the solid.

9. Find the arc length of the parametric curve :

$$x = (1 + t)^2, y = (1 + t)^3, 0 \leq t \leq 1.$$

10. Find the area of the surface generated by revolving the curve $y = \sqrt{4 - x^2}$, $-1 \leq x \leq 1$, about the x-axis.

Section III

Attempt any *three* questions from Section III.

11. Find the equation of the parabola whose focus is $(-1, 4)$ and directrix is $x = 5$.
12. Find the equation of the hyperbola whose foci are $(1, 8)$ and $(1, -12)$ and vertices are 4 units apart.
13. Describe the graph of the equation :

$$9x^2 + 4y^2 + 18x - 24y + 9 = 0.$$