4. (*a*) Find the unique polynomial of degree 2 which fits the given data :

x	0	1	3
$f(x)$	1	3	55

using Lagrange interpolation polynomial. Also estimate value of f at $x = 0.5$ and $x = 2.5$.

(*b*) Prove the following :

$$(1-\nabla)^{-1} = 1+\frac{1}{2}\delta^2+\delta\sqrt{1+\frac{1}{4}\delta^2}.$$

(*c*) Obtain the piecewise linear interpolating polynomials for the function $f(x)$ defined by the given data :

x	0.5	1.5	2.5
$f(x)$	0.125	3.375	15.625

Also find $f(1.0)$ and $f(2.0)$. 13

5. (*a*) Using forward difference formulas, calculate $f'(3)$ and $f''(3)$ from the following data set :

x	1	2	3	4	5
$f(x)$	2	4	8	16	32

(E) 16|5|4

This question paper contains 4+2 printed pages]

Roll No. | | | | | | | | | | | |

57

S. No. of Question Paper : **3103**

Unique Paper Code : **32355402** **IC**

Name of the Paper : **Numerical Methods**

Name of the Course : **Mathematics : Generic Elective for Honours**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All *six* questions are compulsory.

Attempt any *two* parts from each question.

Use of non-programmable scientific calculator is allowed.

1. (*a*) Define Truncation Error. Evaluate the sum $\sqrt{3}+\sqrt{5}+\sqrt{7}$ to four significant digits and find its absolute and relative errors.

(*b*) Perform three iterations of the Bisection Method to obtain a root of the equation :

$$x^3 - 4x - 9 = 0.$$

(*c*) Determine the rate of convergence for the Newton-Raphson Method. 12

2. (*a*) Find a root of the equation :

$$x - e^{-x} = 0$$

correct to three decimal places by the Secant method. Perform three iterations.

(*b*) Perform three iterations using Newton-Raphson method to find a root of the equation :

$$f(x) = x \sin x + \cos x = 0$$

correct to three decimal places, assuming that the root is near $x = \pi$.

(*c*) Perform two iterations of Newton's Method to solve the non-linear system of equations with initial approximation (0.5, 0.5) :

$$f(x, y) = x^2 + 3x + y - 5 = 0$$

$$g(x, y) = x^2 + 3y^2 - 4 = 0.$$ 12

3. (*a*) Find the inverse of the following matrix using the Gauss-Jordan method :

$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}.$$

(*b*) Perform three iterations of Gauss-Seidel method for the following system of equation :

$$-3x + y = -2,$$

$$2x - 3y + z = 0,$$

$$2y - 3z = -1,$$

assuming initial solution as $(x, y, z) = (0, 0, 0)$.

(*c*) Solve the following linear system by using Gaussian elimination with row pivoting :

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 10 \\ 3 & 14 & 28 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ -8 \end{pmatrix}.$$ 12

(*b*) Find an approximate value of the integral

$$I = \int_0^1 e^{-x}\,dx$$

using :

(*i*) Trapezoidal rule

(*ii*) Simpson's rule.

Also calculate the error in each case.

(*c*) Apply Euler's Method to approximate the solution of the initial value problem

$$\frac{dy}{dx} = 1 + \frac{y}{x}, \quad 1 \le x \le 6, \quad y(1) = 1$$

using 5 steps. 13

6. (*a*) Apply mid-point method (second order Runge-Kutta method) to solve the initial value problem using $h = 0.5$:

$$\frac{dy}{dx} = x + y, \quad 0 \le x \le 1$$

$$y(0) = 2.$$

(*b*) Solve the initial value problem using Heun Method (modified Euler Method) :

$$\frac{dx}{dt} = 1 + \frac{x}{t}, \quad 1 \le x \le 2$$

$$x(1) = 1, \qquad h = 1.$$

(*c*) Apply Gaussian Quadrature two point formula to approximate the value of the integral :

$$\mathrm{I} = \int_1^2 \frac{2x}{1+x^4}\,dx.$$ 13