(*b*) Suppose L : $\mathbf{R}^2 \to \mathbf{R}^2$ is a linear operator and L([1, 1]) = [1, –3] and L([–2, 3]) = [–4, 2]. Express L([1, 0]) and L([0, 1]) as linear combinations of the vectors [1, 0] and [0, 1]. 6

(*c*) Let L : $\mathbf{R}^3 \to \mathbf{R}^2$ be the linear transformation given by :

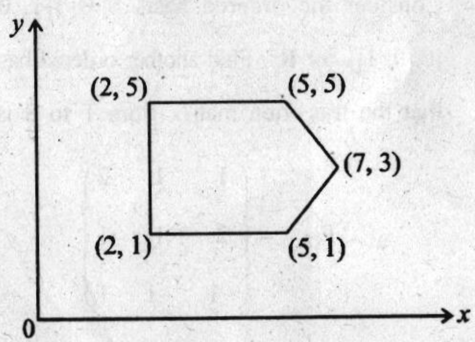
$$L([x, y, z]) = [-2x + 3z, x + 2y - z]$$

Find the matrix for L with respect to the bases :

B = {[1, –3, 2], [–4, 3, –3], [2, –3, 2]} for $\mathbf{R}^3$

and C = {(–2, –1], [5, 3]} for $\mathbf{R}^2$. 6

5. (*a*) For the graphic figure below, use homogeneous coordinates to find the new vertices after performing a scaling about the point (3, 3) with scale factors of 3 in the *x*-direction and 2 in the y-direction. Then sketch the final figure that would result from this movement : 4+2½



11/5/2019 (Evening)

This question paper contains **4+2** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **2980**

Unique Paper Code : **32355202** **IC** 38

Name of the Paper : **Linear Algebra**

Name of the Course : **Generic Elective—Mathematics for Honours**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *all* questions by selecting any *two* parts from each question.

1. (*a*) If *x* and *y* are vectors in $\mathbf{R}^3$, then prove that :

$$\|x\| - \|y\| \le \|x + y\| \le \|x\| + \|y\|.$$ 6½

(*b*) Let *x* and *y* be nonzero vectors in $\mathbf{R}^3$. If $x \cdot y \le 0$, then prove that :

$$\|x - y\| > \|x\|.$$

Is the converse true ? Justify. 6½

(*c*) Solve the following system of linear equations using the Gauss-Jordan method :

$$2x_1 + x_2 + 3x_3 = 16$$

$$2x_1 + 12x_3 - 5x_4 = 5$$

$$3x_1 + 2x_2 + x_4 = 16$$ 6½

2. (*a*) Define the rank of a matrix and determine the rank of $\begin{pmatrix} 1 & 2 & -2 & -11 \\ 2 & 4 & -1 & -10 \\ 3 & 6 & -4 & -25 \end{pmatrix}$. 6

(*b*) Prove that the matrix $\begin{pmatrix} 7 & 1 & -1 \\ 11 & -3 & 2 \\ 18 & 2 & -4 \end{pmatrix}$ cannot be diagonalized. 6

(*c*) Let V be a vector space over **R**, then for any vector v in V and every nonzero real number a, prove that $av = 0$ if and only if $v = 0$. 6

3. (*a*) Let $S = \left\{ \begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix}, \begin{pmatrix} -2 & -5 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 4 \\ -3 & 4 \end{pmatrix} \right\}$ be a subset of 2×2 real matrices. Use the Simplified Span Method to find a simplified form for the vectors in span(S). Is the set S linearly independent ? Justify. 4½+2

(*b*) Define a basis for a vector space. Show that the set :

$$B = \{[-1, 2, -3], [3, 1, 4], [2, -1, 6]\}$$

is a basis for $\mathbf{R}^3$. 2+4½

(*c*) Using rank, find whether the non-homogeneous linear system $Ax = b$, where

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 2 \\ 0 & 1 & 3 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

has a solution or not. If so, find the solution. 4+2½

4. (*a*) Consider the ordered basis $S = \{[1, 0, 1], [1, 1, 0], [0, 0, 1]\}$ for $\mathbf{R}^3$. Find another ordered basis T for $\mathbf{R}^3$ such that the transition matrix from T to S is : 6

$$P_{S \leftarrow T} = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix}.$$

(*b*) Let L : $\mathbf{R}^3 \to \mathbf{R}^3$ be the linear operator given by :

$$L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{pmatrix} 3 & 1 & -3 \\ 2 & 1 & -1 \\ 2 & 3 & 5 \end{pmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find a basis for ker(L) and a basis for range (L), also verify the dimension theorem. 4+2½

(*c*) Show that a mapping L : $P_2 \to P_2$ given by $L(p(x)) = p(x) + p'(x)$ is an isomorphism, where P_2 is the vector space of all polynomials of degree ≤ 2. 6½

6. (*a*) Let W be the subspace of $\mathbf{R}^3$ whose vectors lie in the plane $3x - y + 4z = 0$. Let $v = [2, 2, -3] \in \mathbf{R}^3$. Find $\text{proj}_{W^\perp} v$, and decompose v into $w_1 + w_2$, where $w_1 \in W$ and $w_2 \in W^\perp$. Is the decomposition unique ? 6

(*b*) For the subspace $W = \{[x, y, z] \in \mathbf{R}^3 : 2x - 3y + z = 0\}$ of $\mathbf{R}^3$, find a basis for W and the orthogonal complement $W^{\perp}$. Also verify that :

$$\dim(W) + \dim(W^{\perp}) = \dim(\mathbf{R}^3).$$ 4+2

(c) If $A = \begin{pmatrix} 1 & -1 \\ 4 & 1 \\ 2 & 3 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 4 \\ 5 \end{pmatrix}, z = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. Find vector v satisfying the inequality :

$$\| Av - b \| \leq \| Az - b \|.$$ 6