

(b) Suppose $L : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ is a linear operator and $L([1, 1]) = [1, -3]$ and $L([-2, 3]) = [-4, 2]$. Express $L([1, 0])$ and $L([0, 1])$ as linear combinations of the vectors $[1, 0]$ and $[0, 1]$. 6

(c) Let $L : \mathbf{R}^3 \rightarrow \mathbf{R}^2$ be the linear transformation given by :

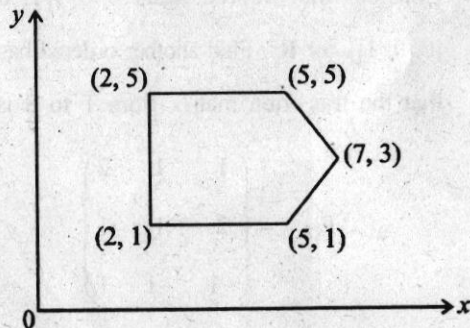
$$L([x, y, z]) = [-2x + 3z, x + 2y - z]$$

Find the matrix for L with respect to the bases :

$$B = \{[1, -3, 2], [-4, 3, -3], [2, -3, 2]\} \text{ for } \mathbf{R}^3$$

$$\text{and } C = \{[-2, -1], [5, 3]\} \text{ for } \mathbf{R}^2. \quad 6$$

5. (a) For the graphic figure below, use homogeneous coordinates to find the new vertices after performing a scaling about the point $(3, 3)$ with scale factors of 3 in the x -direction and 2 in the y -direction. Then sketch the final figure that would result from this movement : 4+2½



11/5/2019 (Evening)
This question paper contains 4+2 printed pages]

Roll No.

--	--	--	--	--	--	--	--	--	--

S. No. of Question Paper : 2980

Unique Paper Code : 32355202

IC

38

Name of the Paper : Linear Algebra

Name of the Course : Generic Elective—Mathematics for
Honours

Semester : II

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting any two parts

from each question.

1. (a) If x and y are vectors in \mathbf{R}^3 , then prove that :

$$\|x\| - \|y\| \leq \|x + y\| \leq \|x\| + \|y\|. \quad 6\frac{1}{2}$$

(b) Let x and y be nonzero vectors in \mathbf{R}^3 . If $x \cdot y \leq 0$, then prove that :

$$\|x - y\| > \|x\|.$$

Is the converse true ? Justify.

6½

P.T.O.

- (c) Solve the following system of linear equations using the Gauss-Jordan method :

$$2x_1 + x_2 + 3x_3 = 16$$

$$2x_1 + 12x_3 - 5x_4 = 5$$

$$3x_1 + 2x_2 + x_4 = 16$$

6½

2. (a) Define the rank of a matrix and determine the rank

of
$$\begin{pmatrix} 1 & 2 & -2 & -11 \\ 2 & 4 & -1 & -10 \\ 3 & 6 & -4 & -25 \end{pmatrix}$$
.

6

- (b) Prove that the matrix
$$\begin{pmatrix} 7 & 1 & -1 \\ 11 & -3 & 2 \\ 18 & 2 & -4 \end{pmatrix}$$
 cannot be

diagonalized.

6

- (c) Let V be a vector space over \mathbf{R} , then for any vector v in V and every nonzero real number a , prove that $av = 0$ if and only if $v = 0$.

6

3. (a) Let $S = \left\{ \begin{pmatrix} 1 & 3 \\ -2 & 1 \end{pmatrix}, \begin{pmatrix} -2 & -5 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 4 \\ -3 & 4 \end{pmatrix} \right\}$ be a subset

of 2×2 real matrices. Use the Simplified Span Method to find a simplified form for the vectors in $\text{span}(S)$. Is the set S linearly independent? Justify. $4\frac{1}{2}+2$

- (b) Define a basis for a vector space. Show that the set :

$$B = \{[-1, 2, -3], [3, 1, 4], [2, -1, 6]\}$$

is a basis for \mathbf{R}^3 . $2+4\frac{1}{2}$

- (c) Using rank, find whether the non-homogeneous linear system $Ax = b$, where

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & -2 & 2 \\ 0 & 1 & 3 \end{pmatrix}, b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

has a solution or not. If so, find the solution. $4+2\frac{1}{2}$

4. (a) Consider the ordered basis $S = \{[1, 0, 1], [1, 1, 0], [0, 0, 1]\}$ for \mathbf{R}^3 . Find another ordered basis T for \mathbf{R}^3 such that the transition matrix from T to S is :

$$P_{S \leftarrow T} = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ -1 & -1 & 1 \end{pmatrix}$$

6