6. (*a*) (*i*) Classify the following partial differential equation into elliptic, parabolic or hyperbolic : 2.5

$$x(xy-1)r-(x^2y^2-1)s+y(xy-1)t+(x-1)p+(y-1)q=0$$

where $r=\frac{\partial^2 z}{\partial x^2}, s=\frac{\partial^2 z}{\partial x \partial y}, t=\frac{\partial^2 z}{\partial y^2}, p=\frac{\partial z}{\partial x}, q=\frac{\partial z}{\partial y}$.

(*ii*) Form a partial differential equation by eliminating constants a, b from the relation : 4

$$z=ax+by+cxy.$$

(*b*) Find the general solution of the differential equation : 6.5

$$x^2(y-x)q+y^2(x-y)p=z(x^2+y^2).$$

(*c*) Find the complete integral of 6.5

$$px+qy=pq.$$

This question paper contains **4** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7168**

Unique Paper Code : **62357502** **J**

Name of the Paper : **Differential Equations**

Name of the Course : **B.A. (Prog.) Mathematics : DSE-2**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All questions are compulsory.

Attempt any *two* parts from each question.

1. (*a*) Solve the initial value problem : 6

$$(y+\sqrt{x^2+y^2})\,dx-x\,dy=0;\ y(1)=0.$$

(*b*) Solve : 6

$$(xy^2-x^2)\,dx+(3x^2y^2+x^2y-2x^3+y^2)\,dy=0$$

(*c*) Solve : 6

$$y+px=x^4p^2$$

2. (*a*) Solve : 6.5

$$\frac{d^2y}{dx^2}+4=\cos 2x+\sin 2x$$

(*b*) Solve : 6.5

$$x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\,(x+\frac{1}{x})$$

(*c*) Consider the differential equation : 6.5

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0$$

(*i*) Verify that $y_1 = e^x$ and $y_2 = e^{2x}$ are the solutions of the above differential equation.

(*ii*) Find a particular solution of the form

$$y = c_1y_1 + c_2y_2$$

that satisfies the initial condition $y(0) = 1$, $y'(0) = 0$.

3. (*a*) Using the method of variation of parameters, solve : 6

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x\,e^x\log x,\quad (x>0)$$

(*b*) Given that $y = x + 1$ is a solution of differential equation : 6

$$(x+1)^2\frac{d^2y}{dx^2}-3(x+1)\frac{dy}{dx}+3y=0\,.$$

Find a linearly independent solution by reducing the order and write the general solution.

(*c*) Solve : 6

$$x^2\frac{d^2y}{dx^2}+2x\frac{dy}{dx}-2y=x^2\log x+3x$$

4. (*a*) Solve the following system of equations : 6.5

$$\frac{dx}{dt}+\frac{dy}{dt}+2x+y=0\,,$$

$$\frac{dy}{dt}+3y+5x=0\,.$$

(*b*) Solve : 6.5

$$\frac{dx}{1}=\frac{dy}{2}=\frac{dz}{5z+\tan(y-2x)}\,.$$

(*c*) Solve : 6.5

$$(ydx+xdy)\,(a-z)+xydz=0\,.$$

5. (*a*) Eliminate the arbitrary function f from the equation : 6

$$z=e^{ax+by}f(ax-by)$$

to find the corresponding partial differential equation.

(*b*) Find the general solution of the differential equation : 6

$$x(y^2-z^2)q-y(x^2+z^2)p=(x^2+y^2)z\,.$$

(*c*) Find the complete integral of the partial differential equation : 6

$$16p^2z^2+9q^2z^2+4z^2-4=0\,.$$