given that $\{Z_t\}$ is a discrete-time, purely random process, such that $E(Z_t)=0$, $V(Z_t)=\sigma^2$ and successive values of Z_t are independent. 10,5

(a) Decomposition of a Time Series,

(b) Harmonic Analysis,

(c) Box-Jenkin's Procedure. 7½, 7½

10/12/18 (M)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	**: 778**	**I**
Unique Paper Code	**: 32377905**	
Name of the Paper	**: Time Series Analysis**	
Name of the Course	**: B.Sc. (H) Statistics : DSE-2**	
Semester	**: V**	
Duration	**: 3 hours**	
Maximum Marks	**: 75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five questions.

1. (a) What is meant by a time series? Illustrate application of a time series from economic and geographic fields with the help of examples.

   (b) Describe the various components of a time series with suitable illustrations. 5, 10

2. In the usual notations, prove that :

$$\frac{1}{m}[m]U_0 = \left[U_0 + \frac{m^2-1}{24}\delta^2 U_0\right]$$

where $\frac{1}{m}[m]$ stands for the simple average of m terms. Further, show that:

$$\frac{1}{m_1 m_2 \ldots m_r}[m_1][m_2]\ldots[m_r]U_0 =$$

5. (a) Suppose that a time series U_t can be represented as the sum of its functional part and random component as:

$$U_t = a_0 + a_1 t + \cdots + a_{k-1} t^{k-1} + \varepsilon_t \;; \quad t = 1, 2, \ldots, n$$

where ε_t's are i.i.d. $N(0, V)$. Show that the estimate of V depends on k.

(b) Show that for an auto-regressive model of order two:

$$U_{t+2} + aU_{t+1} + bU_t = \varepsilon_{t+2},$$

where $|b| < 1$ and ε_t's are i.i.d. $N(0, \sigma^2)$,

$$U_t = \sum_{j=0}^{\infty} \xi_j \varepsilon_{t-j+1}.$$

where $\xi_t = 2p^t \sin(t\theta)/\sqrt{4b - a^2}$. 6,9

6. (a) Explain the Exponential Smoothening procedure for the purpose of forecasting in a time series.

(b) For a moving average of extent m, with weights $(a_1 a_2 \ldots a_m)$ of random components (ε_i; $i = 1, 2, \ldots$), the generated series is given by:

$$U_i = a_1 \varepsilon_{i+1} + a_2 \varepsilon_{i+2} + \cdots + a_m \varepsilon_{i+m} \;;$$

ε_i's are i.i.d. $N(0, \sigma^2)$. Show that:

$$r_k = \begin{cases} \dfrac{\sum_{j=1}^{m-k} a_j a_{j+k}}{\sum_{j=1}^{m} a_j^2} & \text{for } k < m \\ 0 & \text{for } k \geq m \end{cases}$$

8,7

7. Write notes on any *two* of the following:

$$\left[U_0 + \frac{{m_1}^2 + {m_2}^2 + \cdots + {m_r}^2 - r}{24} \delta^2 U_0 \right]$$

Hence deduce Spencer's 15-point formula. 15

3. (a) State the Gompertz curve. Explain the methods of fitting this curve.

(b) Let a time series be composed of only trend and cyclic components, besides the random component. Discuss the effect of eliminating the trend component on the random component of the series. 10, 5

4. (a) Explain seasonal fluctuations in a time-series. How do they differ from cyclic fluctuations? Describe the Link Relative method for measuring the seasonal variations.

(b) Show that the autocorrelation function of the second order MA process

$$X_t = Z_t + 0.7\, Z_{t-1} - 0.2\, Z_{t-2}$$

is given by

$$\rho_k = \begin{cases} 1 & k = 0 \\ 0.37 & k = \pm 1 \\ -0.13 & k = \pm 2 \\ 0 & \text{otherwise} \end{cases}$$

P. T. O.