.10/5/16

This question paper contains **3** printed pages]

**Roll No.** | | | | | | | | | | |

S. No. of Question Paper : **7133**

Unique Paper Code : **2371601** **F-6**

Name of the Paper : **Statistical Inference II**

Name of the Course : **ERSTWHILE FYUP B.Sc. (H) Statistics**

Semester : **VI**

Duration : **Three Hours** Maximum Marks : **75**

*(Write your Roll No. on the top immediately on receipt of this question paper.)*

Attempt any *five* questions.

1. (*a*) What is a statistical hypothesis ? Define :

   (*i*) Two types of errors,

   (*ii*) Power of a test,

   (*iii*) Simple and composite hypotheses, with illustrations.

   (*b*) In a Bernoulli distribution with parameter $p$, $H_0 : p = ½$ against $H_1 : p = ¾$ is rejected, if more than 4 heads are obtained out of 6 throws of a coin. Find the probabilities of type I and type II errors and the power of the test.

   (*c*) Let $X_1, X_2, \ldots\ldots\ldots\ldots, X_n$ be a random sample from discrete distribution with probability function $f(x)$ for which X takes non-negative intergral values 0, 1 2,.................

   Under $H_0 : f(x) = \begin{cases} \dfrac{e^{-1}}{x!}, & x = 0, 1, 2, \ldots\ldots\ldots \\ 0, & \text{otherwise} \end{cases}$

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and under $H_1 : f(x) = \begin{cases} \frac{1}{2^{x+1}}; & x = 0, 1, 2, \ldots\ldots \\ 0\ , & \text{otherwise} \end{cases}$

Obtain the best critical region of size $\alpha$ for testing $H_0$ against $H_1$ for sample of size $n$ and a positive number $k$. Hence, obtain the power of the test for the case $n = 1$ and $k = 1$. 5,5,5

2. (*a*) Explain the concept of most powerful test. State and prove the theorem used to determine the best critical region for testing a simple null hypothesis against a simple alternative hypothesis.

(*b*) Given a random sample $X_1, X_2, \ldots\ldots, X_n$ of size $n$ from the distribution with p.d.f. $f(x, \theta) = \theta e^{-\theta x}; x > 0, 0 < \theta < \infty$.

Show that UMP test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta < \theta_0$ is given by $\left\{ x \mid \sum x_i \geq \frac{1}{2\theta_0} \chi^2_{\alpha,\ 2n} \right\}$, where $\chi^2_{\alpha,\ 2n}$ is the upper $\alpha$-point of the $\chi^2$ distribution with $2n$ degrees of freedom. Also, obtain the power function of the test. 10,5

3. Construct likelihood ratio test for testing $H_0 : \mu = \mu_0$ against various alternatives in case of a random sample of size $n$ drawn from a normal population with mean $\mu$ and known variance $\sigma^2$. 15

4. Describe Wald's S.P.R.T. and its O.C. and A.S.N. functions.

Construct S.P.R.T. for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1 (0 < \theta_0 < \theta_1)$ on the basis of a random sample drawn from Pareto distribution with density function $f(x, \theta) = \frac{\theta a^\theta}{x^{\theta+1}}, x \geq a$. Also obtain its O.C. and A.S.N. functions. 15