

This question paper contains 3 printed pages.

Your Roll No. ....

Sl. No. of Ques. Paper : 1388  
Unique Paper Code : 2371501  
Name of Paper : Statistical Inference - I  
Name of Course : B.Sc. (Hons.) Statistics (Erstwhile FYUP)  
Semester : V  
Duration : 3 hours  
Maximum Marks : 75

F-7

25/11/12 morning

Friday

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any five questions.

1. (a) Let  $X_1, X_2, \dots, X_n$  be a random sample from  $N(\mu, \sigma^2)$ . Examine whether:

$$T = \frac{1}{n} \sum_i |X_i - \mu|$$

is unbiased for  $\sigma$ . If not, obtain an unbiased estimator of  $\sigma$ . Also find efficiency of this unbiased estimator.

- (b) State and prove Cramer-Rao inequality. Under what conditions does equality hold? Explain its significance. 7,8

2. (a) Let  $X_1, X_2, \dots, X_n$  be a random sample from exponential distribution with p.d.f.:

$$f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}, \quad x > 0, \theta > 0$$

Obtain MVB estimator for  $\theta$ . Hence find the variance of MVB estimator.

- (b) State and prove sufficient conditions for consistency. In a random sample of size  $n$  from  $N(\mu, \sigma^2)$ , obtain consistent estimator of  $\sigma^2$  when  $\mu$  is known. 6,9

3. (a) State and prove Factorization theorem for the existence of sufficient statistic.

What is the advantage of this criterion over Fisher-Neyman criterion?

- (b) Let  $Y_1 < Y_2 < \dots < Y_n$  be the order statistics of a random sample of size  $n$  from the uniform distribution having p.d.f. :

$$f(x, \theta) = \begin{cases} \frac{1}{\theta}, & 0 \leq x \leq \theta \\ 0, & \text{otherwise} \end{cases}$$

Show that  $Y_n$  is complete sufficient for  $\theta$ . Hence obtain MVU estimator for  $\theta$ . 7,8

- (a) Show that MVU estimator is unique. Let  $T_1$  and  $T_2$  be two unbiased estimators for  $\theta$  with variances  $\sigma_1^2$  and  $\sigma_2^2$  (both known) and correlation coefficient  $\rho_\theta$ . For what value of  $\alpha$  does

$$T = \alpha T_1 + (1 - \alpha) T_2$$

have minimum variance? Find the variance of  $T$ .

- (b) Explain the method of minimum Chi-square and modified minimum Chi-square. Under what conditions is it identical with the method of maximum likelihood estimation? 8,7

5. (a) Describe method of moments and find estimator of  $\theta$  by the method of moments for:

$$f(x, \theta) = \begin{cases} \frac{1}{2} e^{-|x-\theta|}, & -\infty < x < \infty \\ 0, & \text{otherwise} \end{cases}$$

- (b) Explain the procedure of estimating the parameters by the method of maximum likelihood. Also mention all the properties of ML estimators. 7,8

6. (a) If  $X_1, X_2$  is a random sample of size 2 from a distribution having p.d.f. :

$$f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}, \quad 0 < x < \infty,$$

show that  $Y_1 = X_1 + X_2$  is sufficient estimator for  $\theta$ . Further show that  $Y_2 = X_2$  is an unbiased estimator for  $\theta$  with variance  $\theta^2$ . Find  $E(Y_2 | Y_1 = y_1)$  and compare its variance with that of  $Y_2$ .

- (b) In sampling from a Power Series distribution with probability function:

$$f(x, \theta) = \frac{a_x \theta^x}{\phi(\theta)}, \quad x = 0, 1, 2, \dots$$

where  $a_x$  may be zero for some  $x$ . Show that ML estimator of  $\theta$  is the root of the equation:

$$\bar{x} = \frac{\theta \phi'(\theta)}{\phi(\theta)} = \mu(\theta) \quad \text{or} \quad \mu(\theta) = \bar{x} \quad \text{8,7}$$

7. (a) Distinguish between point estimation and interval estimation. Let  $X_1, X_2, \dots, X_n$  be a random sample of size  $n$  from rectangular distribution with p.d.f.

$$f(x, \theta) = \begin{cases} \frac{1}{\theta}, & 0 \leq x \leq \theta \\ 0, & \text{otherwise} \end{cases}$$