Find (i) significance level of the test, (ii) power of the test.

$(6\frac{1}{2}, 6)$

7. (a) Given a random sample from a population with pdf:

$$f(x, \theta) = \frac{1}{\theta}, \quad 0 \leq x \leq \theta.$$

Show that 100(1-α)% CI for θ is given by R and R/ψ, where ψ is given by $\psi^{n-1}[n - (n-1)\psi] = \alpha$, and R is the sample range.

(b) Obtain 100(1−α)% confidence limits (for large samples) for the parameter λ of the Poisson distribution:

$$f(x, \lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad x = 0, 1, 2, \ldots\ldots \quad (6, 6\tfrac{1}{2})$$

8. Write short notes on any *three* of the following:

i. Optimum properties of ML estimators

ii. Method of moments

iii. Optimum regions and sufficient statistics

iv. LR test and its properties. $(4, 4, 4\frac{1}{2})$

11/5/18 (Morning)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **6722**	**HC**
Unique paper code	: **32371401**	
Name of the paper	: **Statistical Inference**	
Name of course	: **B.Sc. (Hons.) Statistics**	
Semester	: **IV**	
Duration	: **3 hours**	
Maximum marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt **six** questions by selecting **three** from each Section.*

Section I

1. (a) State and prove sufficient conditions for consistency. Give an example where these conditions hold. Are consistent estimators unique? Justify your answer.

(b) If $X_1, X_2, \ldots\ldots, X_n$ constitute a random sample from uniform population U(0, β), show that $T_1 = (\frac{n+1}{n})\, X_{(n)}$ is unbiased estimate for β. Find another unbiased estimate T_2, (say) for β and compare efficiency of T_2 relative to T_1.

$(7, 5\frac{1}{2})$

2. (a) State and prove Rao-Blackwell theorem. Explain its significance in statistical inference.

P. T. O.

(b) Let T_1 and T_2 be unbiased estimators of $\Upsilon(\theta)$ with efficiencies e_1 and e_2 respectively and ρ_θ be the correlation coefficient between them. Then show that:

$$\sqrt{e_1e_2} - \sqrt{(1-e_1)(1-e_2)} \leq \rho_\theta \leq \sqrt{e_1e_2} + \sqrt{(1-e_1)(1-e_2)}$$

$(6\frac{1}{2}, 6)$

3. (a) Show that $Y=X_{(1)}$ the first order statistic of a random sample of size n from distribution with pdf:

$$f(x, \theta) = e^{-(x-\theta)}, \quad \theta < x < \infty$$

is complete sufficient for θ. Using Lehmann Scheffe theorem, find MVUE for θ.

(b) Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from the double Laplace distribution:

$$f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}, \quad -\infty < x < \infty$$

Obtain estimate of θ by using (i) Method of moments, (ii) Method of MLE. $(7\frac{1}{2}, 5)$

4. (a) Describe the method of minimum chi square and show that for large n, estimates obtained by minimum chi square are identical with ML estimates. When do you use modified minimum chi square?

(b) State Cramer-Rao inequality. Explain its significance. Under what conditions does inequality become equality?

(c) Let $X_1, X_2, \ldots\ldots, X_n$ be random sample from $N(\mu, \sigma^2)$ where μ is known. Obtain MVBE for σ^2. $(5, 4, 3\frac{1}{2})$

Section II

5. (a) Define MP critical region and UMP critical region. Let $X_1, X_2, \ldots\ldots, X_n$ be a random sample from a distribution that has pdf:

$$f(x, \theta) = \theta x^{\theta-1}, \quad 0 < x < 1$$

where $0 < \theta < \infty$. Using NP lemma, show that MP test of size α for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ $(\theta_1 > \theta_0)$ is given by the critical region

$$\{(x_1, x_2, \ldots, x_n) / \prod_i x_i > \exp[-\chi^2_{1-\alpha,2n}/2\theta_0]\}$$

where $\chi^2_{1-\alpha,2n}$ is the lower α point of the χ^2 distribution with df = $2n$.

(b) If X is a $B(n, \theta)$ r.v. and prior distribution of θ is a Beta distribution with parameters α and β, then show that posterior distribution of θ given $X=x$ is Beta distribution with parameters $(x+\alpha)$ and $(n-x+\beta)$. $(7, 5\frac{1}{2})$

6. (a) Let $X_1, X_2, \ldots, X_n$ be a random sample of size n from $N(\theta, \sigma^2)$ where σ^2 is known. Develop LR test for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta \neq \theta_0$.

(b) Let X have a pdf of the form:

$$f(x, \theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, \quad 0 < x < \infty$$

to test $H_0 : \theta = 2$ against $H_1 : \theta = 4$. Use a sample X_1, X_2 of size 2 and critical region defined to be $w = \{(X_1, X_2) : 9.5 \leq X_1 + X_2 < \infty\}$.

P. T. O