Section B

5.(a) X= r is unique mode of binomial distribution having mean np and variance $np(1-p)$, show that: 6

$$(n+1)p - 1 < r < (n+1)p$$

Find mode of binomial distribution with $p = \frac{1}{2}$ and $n = 7$.

(b) X is a Poisson variate with parameter λ. Show that 6

$$\mu_{r+1} = r\lambda\mu_{r-1} + \lambda\frac{d\mu_r}{d\lambda}, \qquad r = 1, 2, 3, \ldots\ldots$$

Hence find β_1 and β_2.

6.(a) Obtain the Poisson distribution as a limiting case of negative binomial distribution. 6

(b) If X has a uniform distribution in [0, 1], find the p.d.f. of $-2\log_e X$. Identify the distribution also. 6

7.(a) What is a hypergeometric distribution? Find mean and variance of this distribution. 6

(b) If X and Y are independent Gamma variates with parameters μ and ν respectively, show that the variable U=X+Y and Z = $\frac{X}{X+Y}$ are independent and U is a $\gamma(\mu+\nu)$ variate and Z is a $\beta_1(\mu,\ \nu)$variate. 6

8.(a) Let X~N(0, 1), then find the p.d.f. of Y=$\log_e$ X and hence find mean, mode, standard deviation and coefficient of skewness. 6

(b) Define standard Laplace distribution. Find mean deviation about mean. 6

16|05|2018 (morning)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **6721**	**HC**
Unique paper code	: **32371208**	
Name of the paper	: **Probability and Probability Distributions**	
Name of course	: **B.Sc. (Hons.) Statistics**	
Semester	: **II**	
Duration	: **3 hours**	
Maximum marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Question No. 1 is compulsory. From the remaining questions do any **five** questions by selecting at least **two** questions from each Section. Use of simple calculator is allowed.*

1.(a) For what values of A is $E(|X-A|)$ minimum? 1

(b) If $X \sim B\left(3, \frac{1}{3}\right)$ and $Y \sim B\left(5, \frac{1}{3}\right)$ are independently distributed, then what is the distribution of $X+Y$? 1

(c) Give an example of a continuous distribution for which mean<variance. 1

(d) If X and Y are mutually independent random variables, then evaluate: 1

$$E(XY + Y + 1) - E(X+1)E(Y).$$

(e) From the following table of joint distribution of random variables X and Y: 1

X→ / Y↓	0	1
0	2/9	1/9
1	1/9	5/9

examine whether X and Y are independent.

(f) Let $p(x)$ be the probability function of a discrete random variable X which assume values x_1, x_2, x_3, x_4 such that $2p(x_1)=3p(x_2)=p(x_3)=5p(x_4)$. Find the probability distribution of X. 2

(g) If for a Poisson variate X, $E(X^2)=6$, what is E(X) ? 2

(h) Identify the variate whose m.g.f. is given by: 2

$$M_X(t) = \frac{1}{32}(1+e^t)^5 e^{-2(1-e^t)}$$

(i) If $X \sim N(0,1)$, then find $E(|X|)$. 2

(j) Under what conditions does binomial distribution tend to normal distribution? 2

Section A

2.(a) Let X be a random variable with c.d.f. given by: 6

$$F(x) = \begin{cases} 0, & x \le 0 \\ \frac{x}{2}, & 0 \le x < 1 \\ \frac{1}{2}, & 1 \le x < 2 \\ \frac{x}{4}, & 2 \le x < 4 \\ 1, & x \ge 4 \end{cases}$$

Is the above distribution function continuous? If so, give formula for its p.d.f.

Also find (i) P(X>3), (ii) P(X<3), (iii) P(1<X<3), (iv) P(X>3 | X>1).

(b) Show that for the exponential distribution 6

$$dF(x) = y_0 e^{-x/\sigma} dx, 0 \le x < \infty, \sigma > 0$$

the mean and standard deviation are both equal to σ and that interquartile range is $\sigma \log_e 3$. Also find μ_r' and show that $\beta_1 = 4$, $\beta_2 = 9$.

3.(a) Let X be a r.v. with p.d.f.: 6

$$f(x) = \begin{cases} \frac{2}{9}(x+1), & -1 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$$

Find the p.d.f. of $U=X^2$.

(b) A box contains a white and b black balls. c balls are drawn at random. Find the expected value of the number of white balls drawn. 6

4.(a) The joint density function of a two-dimensional random variable (X, Y) is given by: 6

$$f(x,y) = \begin{cases} 2, & 0 < x < 1, 0 < y < x \\ 0, & \text{elsewhere} \end{cases}$$

(i) Find the marginal density functions of X and Y.

(ii) Find the conditional density function of Y given X= x and conditional density function of X given Y= y.

(iii) Check for independence of X and Y.

(b) Let the random variable X assume the value r with the probability law $P(X=r) = q^{r-1}p$, $r=1, 2, 3, \ldots$. Find the m.g.f. of X and hence the mean and variance. 6