(a) Multicollinearity

(b) Orthogonal columns of x matrix

(c) Partial F test

(d) Residual Analysis. (4,4½,4)

17/05/18 (Morning)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **6723**	**HC**
Unique paper code	: **32371402**	
Name of the paper	: **Linear Models**	
Name of course	: **B.Sc. (Hons.) Statistics**	
Semester	: **IV**	
Duration	: **3 hours**	
Maximum marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt six questions in all, selecting three questions from each Section.

SECTION I

1. (a) What is a parametric function? Derive a necessary and sufficient condition for which a parametric function is estimable. Consider the model $E(Y_1) = 2\beta_1 - \beta_2 - \beta_3$, $E(Y_2) = \beta_2 - \beta_4$, $E(Y_3) = \beta_2 + \beta_3 - 2\beta_4$ with usual assumptions. Find the condition under which $l_1\beta_1 + l_2\beta_2 + m_1\beta_3 + m_2\beta_4$ is an estimable function.

(b) Suppose y_i (i = 1, 2, …, n) is a random sample from a standard normal distribution. Show that $\sum_{i=1}^{n} y_i$ and $\sum_{i=1}^{n}(y_i - \bar{y})^2$ are independently distributed. (7,5½)

P. T. O.

2. For a given model $\underset{\sim}{Y}_{n\cdot1} = X_{n\cdot p}\underset{\sim}{\beta}_{p\cdot1} + \underset{\sim}{\in}_{n\cdot1}$ with $E(\underset{\sim}{\in}) = \underset{\sim}{0}, V(\underset{\sim}{\in}) = \sigma^2 I$ and $\rho(X) = r < p$, prove that the least squares estimator of linear parametric estimable function $c'\underset{\sim}{\beta}$ is BLUE. Also obtain an unbiased estimator of σ^2. (12½)

3. Derive the analysis of covariance for a single factor experimental design with one covariate. (12½)

4. (a) Suppose $\underset{\sim}{Y} \sim N_3(\underset{\sim}{0}, I)$ and let

$$A = \frac{1}{3}\begin{bmatrix} 2 & 0 & -\sqrt{2} \\ 0 & 3 & 0 \\ -\sqrt{2} & 0 & 1 \end{bmatrix}$$

Find the distribution of $\underset{\sim}{Y}'A\underset{\sim}{Y}$, stating the appropriate theorem to be used and also find the distribution of $\underset{\sim}{Y}'D\underset{\sim}{Y}$ and $Y'Y$, where $D = I - A$. Are $\underset{\sim}{Y}'A\underset{\sim}{Y}$ independent of $\underset{\sim}{Y}'D\underset{\sim}{Y}$?

(b) Suppose the hypothesis of homogeneity of k-treatment means is rejected in ANOVA testing for one way classification under fixed effect model, how would you proceed to test the hypothesis of equality of two specific treatment means? (7½,5)

SECTION II

5. (a) For a simple linear regression model $Y_i=\beta_0+\beta_1 X_i+\epsilon_i$, find the 100(1- α)% confidence interval for the difference $\beta_1 - \beta_0$.

(b) Suppose the postulated model is $E(Y)=\beta_0+\beta_1 x_1$ but the true model is $E(Y)=\beta_0+\beta_1 x_1 +\beta_2 x_2$. Show that both $\hat{\beta}_0$ and $\hat{\beta}_1$ are biased by an amount that depends on the values of x's. (6,6½)

6. (a) Write the simple linear regression model in matrix notation. Hence obtain the least squares estimators of the unknown parameters and their variances.

(b) Suppose that we are fitting a straight line and wish to make the standard error of the slope as small as possible. Suppose that the "region of interest" for x is $-1 \le x \le 1$. Where should the observations $x_1, x_2, \ldots, x_n$ be taken? (7½ ,5)

7. (a) For the general linear model, set the appropriate hypothesis for testing the significance of regression and develop the test for significance of individual regression coefficients. What do you mean by global test of model adequacy ?

(b) Consider the simple linear regression model

$$Y = \beta_0 + \beta_1 x + \in \text{ with } E(\in) = 0, V(\in) = \sigma^2,$$

∈'s are uncorrelated. Show that:

$$\text{cov}(\bar{y}, \hat{\beta}_1) = 0$$

$$\text{cov}(\hat{\beta}_0, \hat{\beta}_1) = -\frac{\sigma^2\bar{x}}{S_{xx}}.$$

(7½ ,5)

8. Write a short notes on (Any *three*):