7. Solve any *two* of the following differential equations :

(*a*) $x^2 \dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} - 20y = (x+1)^2$

(*b*) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = x^2e^{3x}$

(*c*) $(2x+3)^2 \dfrac{d^2y}{dx^2} - 2(2x+3)\dfrac{dy}{dx} - 12y = 6x$ 6,6

Section IV

8. Solve any *two* of the following partial differential equations :

(*i*) $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

(*ii*) $z^2(p^2 + q^2) = x^2 + y^2$

(*iii*) $p(1 + q^2) = q(z - a)$ 6,6

9. (*a*) Solve :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \cos x \cos 2y.$$

(*b*) Solve the partial differential equation : 6,6

$$(D^2 + DD' - 6D'^2)z = y \cos x.$$

13/12/18 (M)

This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **117**

Unique Paper Code : **32371109** **I**

Name of the Paper : **Calculus**

Name of the Course : **B.Sc. (Hons.) Statistics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. **1** is compulsory.

From the remaining, attempt *five* questions,

selecting at least *one* from each section.

1. Attempt any *five* parts : 3×5

(*a*) Evaluate : $\lim\limits_{h \to 0} \dfrac{[\log_e(1+2h) - 2\log_e(1+h)]}{h^2}$

(*b*) Examine the continuity of the function at $x = 0$:

$$f(x) = \begin{cases} \dfrac{e^{-1/x}}{1+e^{1/x}}; & x \neq 0 \\ 1 \quad ; & x = 0 \end{cases}$$

P.T.O.

(*c*) Compute the value of $\Gamma(-3/2)$.

(*d*) Show that :

$$\int_0^1 \int_0^{x^2} e^{y/x}\, dx\, dy = \frac{1}{2}.$$

(*e*) Solve :

$$\sqrt{(a^2+x^2)}\,\frac{dy}{dx} + y = \sqrt{(a^2+x^2)} - x.$$

(*f*) Solve the differential equation :

$$(4D^2 + 4D - 3)y = e^{2x}.$$

(*g*) By eliminating the constants, obtain the partial differential equation from the relation $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

(*h*) Solve the partial differential equation :

$$z = p^2x + q^2y.$$

Section I

2. (*a*) If $y = x(x+1)\log(x+1)^3$, then prove that

$$\frac{d^n y}{dx^n} = \frac{3(-1)^{n-1}(n-3)!\,(2x+n)}{(x+1)^{n-1}} \quad \text{if } n \geq 3.$$

(*b*) Obtain the maximum or minimum value of u given by $u = x^3y^2(1-x-y)$. 6,6

3. (*a*) If $\theta = t^n e^{-r^2/4t}$, then find the value of n which will make

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}.$$

(*b*) Find the position and nature of the double points on the curve : 6,6

$$y(y-6) = x^2(x-2)^3 - 9.$$

Section II

4. (*a*) Show that :

$$B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$

(*b*) Evaluate : 6,6

$$\int_0^1 \frac{x^3}{(x^2+1)(x^2+7x+12)}\, dx.$$

5. (*a*) Find the limit, when n tends to infinity of the sum

$$\sum_{r=1}^{n-1} \frac{1}{n}\sqrt{\frac{n+r}{n-r}}.$$

(*b*) Change the order of integration in $\int_0^\infty \int_x^\infty \left(\frac{e^{-y}}{y}\right) dx\, dy$ and hence find its value. 6,6

Section III

6. Solve the following differential equations :

(*a*) $(D^3 + 1)y = \cos 2x$

(*b*) $\frac{1}{y}\frac{dy}{dx} + \frac{x}{1-x^2} = xy^{-1/2}$. 6,6