i) Hermitian matrix,
ii) Real symmetric matrix, and
iii) Unitary matrix,

are orthogonal.

(b) Define generalized inverse. Give an algorithm for finding a generalized inverse of a symmetric matrix A. Hence find a g-inverse of :

$$\begin{pmatrix} 2 & 2 & 6 \\ 2 & 3 & 8 \\ 6 & 8 & 22 \end{pmatrix}$$

(6, 6½)

8. (a) Reduce the quadratic form :

$$21x_1^2 + 11x_2^2 + 2x_3^2 - 30x_1x_2 - 8x_2x_3 + 12x_3x_1$$

to the canonical form and hence show that it is positive semi-definite. Also find a non-zero set of values of x_1, x_2, x_3 which make the form zero.

(b) Compute the inverse of the given matrix by the method of partitioning:

$$A = \left(\begin{array}{c|cc} 2 & 1 & 0 \\ \hline 3 & 0 & 5 \\ 7 & 6 & 4 \end{array}\right)$$

(6 ½ , 6)

22/5/18

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **6720**	**HC**
Unique paper code	: **32371202**	
Name of the paper	: **Algebra**	
Name of course	: **B.Sc. (Hons.) Statistics**	
Semester	: **II**	
Duration	: **3 hours**	
Maximum marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt six questions in all, selecting three questions from each Section.

SECTION I

1.(a) Solve the equation

$$x^4 - 18x^3 + 136x^2 - 720x + 1600 = 0,$$

given that the product of two of its roots is equal to the product of the other two.

(b) Form an equation whose roots are the ratios of the roots α , β , γ of the equation $x^3 + qx + r = 0$. (6 ½ , 6)

2. (a) Form a cubic equation whose roots are the values of α , β , γ given by the relation $\alpha + \beta + \gamma = 2$, $\alpha^2 + \beta^2 + \gamma^2 = 6$, $\alpha^3 + \beta^3 + \gamma^3 = 8$. Find the value of $\alpha^4 + \beta^4 + \gamma^4$.

P. T. O.

(b) Show that the set $\{1, x, 1+x+x^2\}$ is a linearly independent set of vectors in the vector space of all polynomials over the real number fields.

(c) Show that the set of vectors $\{(0, 1, 0), (1, 0, 1), (1, 1, 0)\}$ constitutes a basis of the real vector space R^3. (6, 3, 3½)

3. (a) Let A be a skew symmetric determinant given by

$$A = \begin{vmatrix} 0 & x & y & z \\ -x & 0 & c & b \\ -y & -c & 0 & a \\ -z & -b & -a & 0 \end{vmatrix}$$

Show that A can be expressed as a square of a polynomial function of its elements.

b) Express the determinant

$$\begin{vmatrix} 1 & \cos(\beta - \alpha) & \cos(\gamma - \alpha) \\ \cos(\alpha - \beta) & 1 & \cos(\gamma - \beta) \\ \cos(\alpha - \gamma) & \cos(\beta - \gamma) & 1 \end{vmatrix}$$

as a product of two determinants. Hence or otherwise show that it vanishes. (6½ , 6)

4. (a) What are Echelon matrices? Reduce the matrix A to its echelon form and mark the pivot entries.

$$A = \begin{pmatrix} 1 & 2 & 1 & 1 \\ 1 & 2 & 2 & -1 \\ 2 & 4 & 0 & 6 \end{pmatrix}$$

(b) Show that the only real value of λ for which the following system of linear equations has non zero solutions is 6 and then solve the equations:

$$x + 2y + 3z = \lambda x$$
$$3x + y + 2z = \lambda y$$
$$2x + 3y + z = \lambda z$$

(6 , 6 ½)

SECTION II

5. (a) A is a square matrix of order n with all elements equal to unity and B is a square matrix of order n with all diagonal elements equal to n and other elements n−r. Show that $A^2 = nA$. Deduce that $(B - rI)\,[B - (n^2 - nr + r)\,I\,] = O$.

(b) Find the value of adj $(\mathbf{P}^{-1})$ in terms of **P** where **P** is a non-singular matrix and hence show $\text{adj}(\mathbf{Q}^{-1}\,\mathbf{B}\mathbf{P}^{-1}) = \mathbf{PAQ}$, given that adj $\mathbf{B} = \mathbf{A}$ and $|\mathbf{P}| = |\mathbf{Q}| = 1$. (6, 6½)

6. (a) If A is a square matrix and $(A - \frac{1}{2}I)$ and $(A + \frac{1}{2}I)$ are orthogonal, then prove that A is skew-symmetric and $A^2 = -\frac{3}{4}I$. Deduce that A is of even order.

(b) Find non singular matrices R and S such that RAS is in normal form, where:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

Hence find rank of A. (6, 6½)

7. (a) Show that any two characteristic vectors corresponding to two distinct characteristic roots of a: