$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(2n-1)}$

is conditionally convergent.

(d) Test the following series for Absolute convergence:

$$\sum_{n=1}^{\infty}(-1)^n\frac{(n!)^2}{(2n)!}.$$

(5, 5, 5)

16/05/18 (morning)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: **6630**	**HC**
Unique paper code	: **32351201**	
Name of the paper	: **Real Analysis**	
Name of course	: **B.Sc. (Hons.) Mathematics**	
Semester	: **II**	
Duration	: **3 hours**	
Maximum marks	: **75**	

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any three parts from each question.

All questions are compulsory.

1. (a) Prove that a lower bound v of a nonempty set S in **R** is the Infimum of S if and only if for every $\epsilon > 0$, there exists an $s_\epsilon \in S$ such that $s_\epsilon < v + \epsilon$.

(b) Let S be a nonempty bounded above set in **R**. Let a > 0 and aS = {as: s ∈ S}, then prove that Sup (aS) = a Sup S.

(c) If x and y are positive real numbers with x < y, then prove that there exists a rational number r ∈ Q such that x < r < y.

(d) Show that Sup $\left\{1-\frac{1}{n}: n \in \mathbf{N}\right\} = 1$. 5,5,5

2. (a) Define limit point of a set in **R**. Prove that a point $c \in \mathbf{R}$ is a limit point of a set S if and only if every neighbourhood of c contains infinitely many points of S.

(b) Let (x_n) be a sequence of real numbers such that $\lim_{n\to\infty} x_n = x > 0$, then show that there exists a natural number K such that

$$\frac{x}{2} < x_n < 2x \quad \forall\, n \geq K.$$

(c) Use the definition of limit to prove:

i. $\lim_{n\to\infty} (\sqrt{n+1} - \sqrt{n}) = 0$

ii. $\lim_{n\to\infty} \left(\frac{3n+2}{n+1}\right) = 3$.

(d) Let (x_n) be a sequence of positive real numbers such that $L = \lim_{n\to\infty} \left(\frac{x_{n+1}}{x_n}\right)$ exists. Show that if $L < 1$, then (x_n) converges and $\lim_{n\to\infty} x_n = 0$. (5, 5, 5)

3. (a) Let (x_n) and (y_n) be sequences of real numbers such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$, then show that $\lim_{n\to\infty} x_n y_n = xy$.

(b) State Squeeze Theorem and hence prove that

$$\lim_{n\to\infty} (a^n + b^n)^{1/n} = b,$$

where $0 < a < b$.

(c) State and prove Monotone Convergence Theorem.

(d) Let (x_n) be a sequence of real numbers defined by

$$x_1 = 8, \; x_{n+1} = \frac{x_n}{2} + 2 \quad \text{for } n \in \mathbf{N}.$$

Show that (x_n) is convergent and find its limit. (5, 5, 5)

4. (a) Show that the following sequences are divergent:

(i) $((-1)^n)$

(ii) $\left(\sin\left(\frac{n\pi}{3}\right)\right)$.

(b) Define a Cauchy sequence and show that every Cauchy sequence of real numbers is bounded.

(c) Prove that the sequence (x_n), where

$$x_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}, \qquad n \in \mathbf{N}$$

is not a Cauchy sequence.

(d) Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be infinite series of positive real numbers such that $\lim_{n\to\infty} \left(\frac{a_n}{b_n}\right) = 0$. Show that if $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. (5, 5, 5)

5. (a) State and prove *n*-th Root Test to test the convergence of an infinite series.

(b) Test for convergence any *two* of the following series:

i. $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

ii. $\sum_{n=1}^{\infty} \frac{1}{(2n+1)^2}$

iii. $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$.

(c) Define Conditional Convergence. Show that the series