(b) Using the method of separation of variables, discuss the problem of vibrating string:

$$u_{tt} - c^2 u_{xx} = 0, \quad 0 < x < l, \quad t > 0,$$
$$u(x,0) = f(x), \quad u_t(x,0) = g(x), \quad 0 \le x \le l,$$
$$u(0,t) = u(l,t) = 0, \quad t > 0. \qquad (7)$$

(c) Determine the solution of Initial Boundary Value problem

$$u_{tt} = c^2 u_{xx} + x^2, \quad 0 < x < 1, \quad t > 0,$$
$$u(x,0) = x, \quad u_t(x,0) = 0, \quad 0 \le x \le 1,$$
$$u(0,t) = 0, \quad u(l,t) = 1, \quad t > 0. \qquad (7)$$

(d) Find the solution of the plucked string of length l equation given by:

$$u_{tt} - c^2 u_{xx} = 0$$
$$u(x,0) = 0,$$
$$u_t(x,0) = \begin{cases} \dfrac{v_0 x}{a}, & 0 \le x \le a \\ \dfrac{v_0 (l-x)}{(l-a)}, & a \le x \le l \end{cases}$$
$$u(0,t) = u(l,t) = 0, \quad t > 0. \qquad (7)$$

11/5/18 (morning)

This question paper contains 4 printed pages.

Your Roll No. ....

HC

S. No. of Paper : **6632**

Unique paper code : **32351401**

Name of the paper : **Partial Differential Equations**

Name of course : **B.Sc. (Hons.) Mathematics**

Semester : **IV**

Duration : **3 hours**

Maximum marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

All Sections are compulsory.
Marks of each part are indicated.

SECTION – I

*Attempt any **two** parts out of the following.*

1. (a) Obtain the general solution of the equation

$$(z^2 - 2yz - y^2)p + x(y+z)q = x(y-z).$$

Find the integral surfaces of this equation passing through (i) the x-axis, (ii) the y-axis, (iii) the z-axis. $\left(7\frac{1}{2}\right)$

(b) Solve the following initial value system:

$$u_t + u u_x = e^{-x} v, \quad v_t - a v_x = 0$$

with $u(x,0) = x$ and $v(x,0) = e^x$. $\left(7\frac{1}{2}\right)$

P. T. O.

(c) Apply the method of separation of variables $u(x,y)=f(x)g(y)$ to solve

$$x^2u_{xy}+9y^2u=0 \qquad u(x,0)=\exp\left(\frac{1}{x}\right) \qquad \left(7\frac{1}{2}\right)$$

SECTION – II

*Attempt any **one** part out of the following.*

2. (a) Show that the equation of motion of a one-dimensional wave equation is $u_{tt}=c^2u_{xx}$ where $c^2=\tau/\rho$. (6)

(b) Derive the Laplace equation of motion. (6)

*Attempt any **two** parts out of the following.*

3. (a) Find the characteristics, characteristic coordinates and reduce the equation given below to the canonical form:

$$u_{xx}+(\sec h^4 x)u_{yy}=0. \qquad (6)$$

(b) Transform the following equations to the form:

$v_{\xi\eta}=cv$, c =constant, $u_{xx}-u_{yy}+3u_x-2u_y+u=0$

by introducing the new variable $v=ue^{-(a\xi+b\eta)}$, where a, [illegible] are undetermined coefficients. (6)

(c) Determine the general solution of the equation given below:

$$3u_{xx}+10u_{xy}+3u_{yy}=0 \qquad (6)$$

SECTION – III

*Attempt any **three** parts out of the following.*

4. (a) Determine the solution of the equation:

$$L[u]\equiv u_{xy}+au_x+bu_y+cu=f(x,y)$$

where a,b,c and f are differentiable functions of (x,y) in some domain D^*, under the boundary conditions that u and u_x are prescribed along the curve C in the xy plane. (7)

(b) Solve the Goursat problem

$$xy^3u_{xx}-x^3yu_{yy}-y^3u_x+x^3u_y=0,\ x\neq 0,$$

$$u(x,y)=f(x)\ \textit{on}\ \ y^2+x^2=16\ \ \textit{for}\ \ 0\le x\le 4,$$

$$u(x,y)=g(y)\ \textit{on}\ \ x=0\ \ \textit{for}\ \ 0\le y\le 4,$$

where $f(0)=g(4)$. (7)

(c) Find the solution of the initial boundary value problem:

$$u_{tt}-c^2u_{xx}=0$$

$$u(x,0)=\sin x, \qquad u_t(x,0)=x^2 \qquad (7)$$

(d) Solve the Cauchy problem for the non-homogeneous wave equation

$$u_{tt}=c^2u_{xx}+e^x.$$

with the initial conditions:

$$u(x,0)=5, \qquad u_t(x,0)=x^2. \qquad (7)$$

SECTION – IV

*Attempt any **three** parts out of the following.*

5. (a) Solve using the method of separation of variables:

$$u_t=ku_{xx},\ 0<x<l,\ t>0.$$

$$u(0,t)=0\ ,\ t\ge 0$$

$$u(l,t)=1\ ,\ t\ge 0$$

$$u(x,0)=\sin\frac{\pi x}{2l},\ \ 0\le x\le l. \qquad (7)$$

P. T. O.