

6. (a) Using Trapezoidal rule approximate the value of the integral:

$$\int_0^2 \tan^{-1} x \, dx.$$

Further verify the theoretical error bound.

- (b) Derive the closed Newton-Cotes rule ( $n = 3$ ) for the computation of the definite integral:

$$\int_a^b f(x) \, dx.$$

- (c) Apply Euler's method to approximate the solution of the given initial value problem:

$$x' = \frac{1+x^2}{t}, (1 \leq t \leq 4), x(1) = 0, N = 5.$$

Further it is given that the exact solution is:

$$x(t) = \tan(\ln(t)).$$

Compute the absolute error at each step. 12

10/12/18 (M)

This question paper contains 6 printed pages.

Your Roll No. ....

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 Name of the Course : B.Sc. (H) Mathematics : DSE-2  
 Semester : V  
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(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions, selecting two parts from each question. Use of non-programmable scientific calculator is allowed.

1. (a) Given the following scheme for integration:

$$\int_a^b f(x) \, dx \approx \frac{h}{2} + [f(a) + 2 \sum_{i=1}^{n-1} f(x_i) + f(b)],$$

write an algorithm to obtain the approximate value of the definite integral.

- (b) Verify that the equation  $x^5 - 2x - 1 = 0$  has a root in the interval  $(0, 1)$ . Perform three iterations to approximate the zero of the equation by the Secant method using  $p_0 = 0$  and  $p_1 = 1$ .

- (c) Let  $f$  be a continuous function on the closed interval  $[a, b]$  and suppose that  $f(a)f(b) < 0$ .

Prove that the bisection method generates a sequence of approximations  $\{p_n\}$  which converges to a root  $p \in (a, b)$  with the property

$$|p_n - p| \leq \frac{b-a}{2^n}.$$

Hence, find the rate of the convergence of the method. 13

2. (a) Give the geometrical construction of the method of False Position to approximate the zero of a function. Further, write the algorithm for the computation of the root approximated by this method.

- (b) Perform three iterations for finding the root of

$$f(x) = \frac{1}{x} - 37$$

by Newton's method starting with  $p_0 = 0.01$ .

Further, compute the ratio

$$|p_3 - p| / |p_2 - p|^2$$

and show that this value approaches  $|f''(p) / 2f'(p)|$ , with  $p = 1/37$ .

- (ii) Define the backward difference operator and the central operator. Prove that:

$$\delta = \nabla (1 - \nabla)^{-1/2}. \quad 12$$

5. (a) Derive the formula:

$$f''(x_0) \approx \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2},$$

the second-order central difference approximation to the second order derivative of a function.

- (b) Verify that:

$$f'(x) \approx \frac{f(x_0+h) - f(x_0-h)}{2h},$$

the difference approximation for the first order derivative provides the exact value of the derivative regardless of  $h$ , for the functions  $f(x) = 1$ ,  $f(x) = x$  and  $f(x) = x^2$ , but not for the function  $f(x) = x^3$ .

- (c) Use the formula:

$$f'(x) \approx \frac{f(x_0) - f(x_0 - h)}{h}$$

to approximate the derivative of the function  $f(x) = e^x$  at  $x_0 = 0$ , taking  $h = 1, 0.1, 0.01$  and  $0.001$ . What is the order of approximation? 12