(ii) Write a short note on interest rate swap. 4

(b) (i) Does a perfect hedge always succeed in locking in the current spot price of an asset for a future transaction? Explain your answer. 4

(ii) Explain the difference between selling a call option and buying a put option. 2

(c) An investor buys a European put on a share for \$2.5. The stock price is \$45 and the strike price is \$42. Under what circumstances does the investor make a profit? Under what circumstances will the option be exercised? Draw a diagram showing the variation of the investor's profit with the stock price at the maturity of the option. 6

6. (a) Suppose price of a stock is \$31, exercise price is \$30, risk-free interest rate is 10% per annum, the price of a 3-month European call option is \$3 and the price of a 3-month European put option is \$2.25. Is there put-call parity? Can an arbitrageur make profit at the end of 3 months? Justify. $6\frac{1}{2}$

(b) (i) Draw and explain profit from buying a European call option on one share of a stock. Given option price is \$5 and strike price is \$100. $3\frac{1}{2}$

(ii) Draw and explain payoff from a short call position in a European option with strike price= K, price of asset at maturity = S_T. 3

(c) (i) What is the difference between the forward price and the value of a forward contract? 2

(ii) Suppose that you enter into a 3-month forward contract on a non-dividend-paying stock when the stock price is \$30 and the risk-free interest rate is 10% per annum. What is the forward price? $4\frac{1}{2}$

600

10/12/18 (M)

This question paper contains 4 printed pages.

Your Roll No. ....

S. No. of Paper	: 755 **I**
Unique Paper Code	: **32357504**
Name of the Paper	: **Mathematical Finance**
Name of the Course	: **B.Sc. (H) Mathematics : DSE-2**
Semester	: **V**
Duration	: **3 hours**
Maximum Marks	: 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

*Attempt any **two** parts from each question.*

Following values may be used if needed :

$$e^{0.025} = 1.0253,\ e^{-0.025} = 0.975,$$
$$e^{0.0125} = 1.0125 \text{ and } e^{-0.0125} = 0.9875.$$

1. (a) (i) Define present value and future value of a cash flow stream. $1\frac{1}{2} + 1\frac{1}{2}$

(ii) What are callable bonds? Discuss mortgage. $1 + 2\frac{1}{2}$

(b) (i) Find effective rate for 3% compounded monthly. $2\frac{1}{2}$

(ii) A 10% bond with 20 years to maturity has a yield of 9%. What is the price of this bond? 4

(c) A major lottery advertises that it pays the winner \$10 million. However, this prize money is paid at the rate of \$500,000 each year (with the first payment being immediate) for a total of 20 payments. What is the present value of this prize at 10% interest? $6\frac{1}{2}$

2. (a) State and prove the price sensitivity formula. Consider a 30-year zero coupon bond. Suppose that its current yield is 10%, $D = 30$, $D_M \approx 27$. Suppose that the yields increase to 11%. Find the corresponding change in the price. $6\frac{1}{2}$

(b) In general, we say that two projects with cash flows x_i and y_i, $i = 0,1,2,\ldots,n$, cross if $x_0 < y_0$ and $\sum_{i=0}^{n} x_i > \sum_{i=0}^{n} y_i$. Let $P_x(d)$ and $P_y(d)$ denote the present values of these two projects when the discount factor is d. Show that there is a crossover value $c > 0$ such that $P_x(c) = P_y(c)$. $6\frac{1}{2}$

(c) (i) Define Macaulay duration. 3

(ii) Use Macaulay duration formula to calculate duration of a 7% bond with 3 years to maturity and 8% yield. $3\frac{1}{2}$

3. (a) Suppose that there are two assets with $\bar{r}_1 = 0.10$, $\bar{r}_2 = 0.16$, $\sigma_1 = 0.15$, $\sigma_2 = 0.25$ and $\sigma_{12} = 0.04$. A portfolio is formed with weights $w_1 = 0.30$ and $w_2 = 0.70$. Calculate the mean and the variance of the portfolio. Draw the triangular region within which any point corresponding to a portfolio consisting of non-negative mixture of the two assets could lie. 6

(b) State One Fund Theorem. In the mean-variance portfolio theory:

(i) Discuss the effect of inclusion of a risk-free asset on the efficient frontier.

(ii) Which point on the efficient frontier corresponds to the one fund consisting of risky assets as

specified in the above theorem? Explain. 6

(c) There are just three assets with rates of return r_1, r_2 and r_3, respectively. The covariance matrix and the expected rates of return are :

$$V = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad \bar{r} = \begin{bmatrix} 0.4 \\ 0.8 \\ 0.8 \end{bmatrix}$$

Find the minimum variance portfolio. 6

4. (a) Suppose there are n risky assets. If the market portfolio M is efficient, then prove that the expected return $\bar{r}_i$ of any asset i satisfies:

$$\bar{r}_i - r_f = \beta_i(\bar{r}_M - r_f)$$

where symbols have usual meaning. 6

(b) Mr. S is young and impatient. He notes that the risk-free rate is only 6% and the market portfolio of risky assets has an expected return of 12% and a standard deviation of 15%. He determines that he must attain an average rate of return of about 100% to achieve his goal. What is the expected risk? 6

(c) Assume that the expected rate of return on the market portfolio is 23% and the rate of return on T-bills (the risk free rate) is 7%. The standard deviation of the market is 32%. Assume that the market portfolio is efficient.

(i) What is the equation of the capital market line?

(ii) If you invest \$300 in the risk-free asset and \$700 in the market portfolio, how much money should you expect to have at the end of the year? 6

5. (a) (i) What is the difference between a long forward position and a short forward position? 2