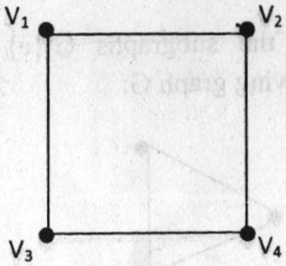
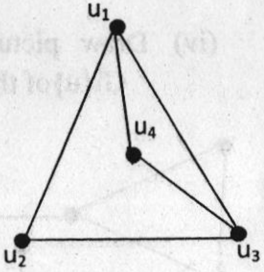
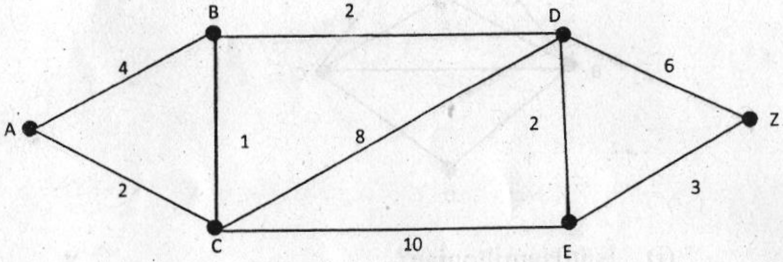


G_1



G_2

(c) Apply the first form of Dijkstra's Algorithm to find a shortest path from A to Z in the graph shown. Label all vertices. 6.5



10/12/18 (M)

This question paper contains 6 printed pages.

Your Roll No. ....

S. No. of Paper	: 756 **I**
Unique Paper Code	: 32357505
Name of the Paper	: **Discrete Mathematics**
Name of the Course	: **B.Sc. (H) Mathematics : DSE-2**
Semester	: V
Duration	: 3 hours
Maximum Marks	: 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions, selecting two parts from each question.

Section I

1. (a) Let P and Q be finite ordered sets and let $\varphi : P \to Q$ be a bijective map. Then prove that the following statements are equivalent:

(i) φ is an order-isomorphism.

(ii) $x < y$ in P if and only if $\varphi(x) < \varphi(y)$ in Q.

(iii) $x \prec y$ in P if and only if $\varphi(x) \prec \varphi(y)$ in Q. 6

(b) Let $\mathbb{N}_0$ be the set of whole numbers equipped with the partial order $\leq$ defined by $m \leq n$ if and only if m divides n. Draw a Hasse diagram for the subset $P = \{2,3,12,18\}$ of $(\mathbb{N}_0, \leq)$. How many maximal & minimal elements are there in $(P, \leq)$?

P. T. O.

Does it have the smallest and the greatest elements? Justify your answer. 6

(c) Give an example of an ordered set having three elements x, y, z such that:

(i) $\{x, y, z\}$ is an antichain.

(ii) $x \vee y, y \vee z$ and $z \vee x$ fail to exist.

(iii) $\vee\{x, y, z\}$ exists. 6

2. (a) Let $(L, \vee, \wedge)$ be an algebraic lattice. Define a relation $\leq$ on L as $x \leq y$ in L if and only if $x \wedge y = x$. Then prove that $(L, \leq)$ is a lattice as an ordered set. 6.5

(b) Prove that every finite lattice is bounded. Give an example of:

(i) a lattice with unit element but without a zero element.

(ii) a lattice having neither a zero element nor the unit element. 6.5

(c) Let f be a monomorphism from a lattice L into a lattice M. Show that L is isomorphic to a sublattice of M. 6.5

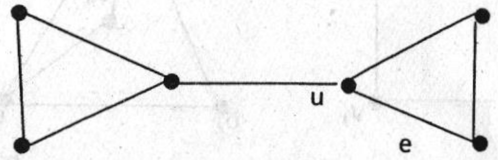
Section II

3. (a) Prove that a lattice L is modular if and only if $\forall a, b, c \in L; a \geq c$, we have:

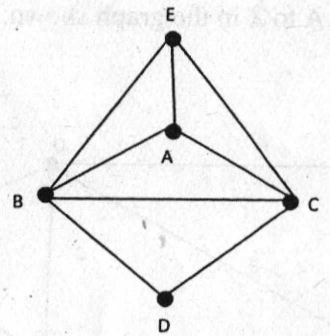
$$(a \vee b = c \vee b \text{ and } a \wedge b = c \wedge b) \Rightarrow a = c.$$

6

(iv) Draw pictures of the subgraphs $G \backslash \{e\}$ and $G \backslash \{u\}$ of the following graph G: 2,2,2



6. (a) Consider the following graph:



(i) Is it Hamiltonian?

(ii) Is there a Hamiltonian Path?

(iii) Is it Eulerian?

(iv) Is there an Eulerian trail?

Explain your answer. 6.5

(b) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown below. Find a permutation matrix P such that $A_2 = PA_1P^T$. 6.5

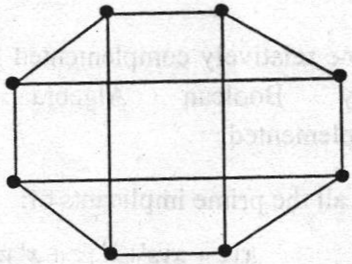
Section III

5. (a) (i) Draw a graph with 6 vertices and as many edges as possible. How many edges does your graph contain? What is the name of this graph and how is it denoted?

(ii) Prove that the number of odd vertices in a pseudograph is even. 3,3

(b) (i) Draw $K_{2,6}$ and $K_{4,4}$.

(ii) What is bipartite graph? Determine whether graph given below is bipartite. Give the bipartition sets or explain why the graph is not bipartite. Is it a complete bipartite graph?

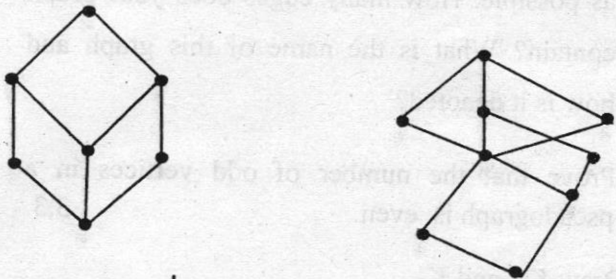


3,3

(c) (i) Does there exist a graph G with 28 edges and 12 vertices, each of degree 3 or 6? Justify your answer.

(ii) Draw a graph whose degree sequence is 1,1,1,1,1,1.

(b) State M_3-N_5 theorem and use it to find if the lattices L_1 and L_2 given below are modular or distributive:



L_1 L_2

6

(c) Find the Disjunctive Normal Form of f and simplify it:

$$f = x'y + x'y'z + xy'z' + xy'z.$$

6

4. (a) Define relatively complemented lattice. Show that every Boolean Algebra is relatively complemented. 6.5

(b) Find all the prime implicants of:

$$xyz + xyz' + x'yz + x'yz'$$

and form the corresponding prime implicant table. 6.5

(c) Draw the contact diagram and using six gates, determine the symbolic representation of the circuit given by:

$$p = (x_1 + x_3)'(x_1' + (x_2 + x_3)(x_2' + x_3')).$$

6.5